

**Argonne National Laboratory**

**THE EDDY-CURRENT FLOWMETER:**

**An Analysis Giving Performance Characteristics  
and Preferred Operating Conditions**

**by**

**David E. Wiegand**

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9700 South Cass Avenue  
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Reactor Engineering Division

August 1969



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## NOMENCLATURE

$a$	Dimensionless variable, proportional to an area (Eq. 62).	$K_n$	Normalization factor for the signal voltage (Eq. 59).
$A$	Area of a circle of radius $r$ (Eq. 52).	$L$	Length of a coil (Eq. 12).
$A, B$	Constants in a velocity-profile relation (Eq. 65).	$L$	Length of a pipe (Eq. 85).
$A_C$	Area of the flow channel (Eq. 54).	$L_g$	Length of a secondary (generator) coil (Eq. 42).
$be$	Complex combination of ber and bei (Eq. 35).	$m$	Exponent in the velocity-profile equation applied to areas (Eq. 78).
$ber, bei,$ $ker, kei$	Kelvin functions of zero order (Eq. 25).	$M$	Mutual inductance (Eq. 1).
$B_r$	Average radial component of magnetic induction at radius $r$ , inside a secondary coil section (Eq. 42).	$n$	Exponent in the velocity-profile equation applied to radii (Eq. 69).
$C_1, C_2$	Arbitrary constants in the solution of the Bessel equation (Eq. 25).	$N_d$	Number of turns in a primary coil section (Eq. 45).
$d_1, d_2,$ $d_3, \dots$	Primary (driver) coil sections (Fig. 3).	$N_{dg}$	Number of primary-secondary coil sections (Eq. 56).
$dber, dbei$	Derivatives of the Kelvin functions (Eq. 82).	$N_g$	Number of turns in a secondary coil section (Eq. 48).
$D_c$	Diameter of the flow channel (Eq. 45).	$q$	Volume rate of the fluid (Eq. 96).
$f$	Frequency of the primary current (Eq. 52).	$r$	Radial distance in the fluid (Eq. 12).
$F$	Force (Eq. 1).	$r_1$	Value of $r$ at limit of integration (Eq. 36).
$F_{sp}$	Signal-profile function (Eq. 57).	$R$	Channel radius (Eq. 80).
$\text{FUNC}_R,$ $\text{FUNC}_I$	Real and imaginary components of a dimensionless variable used in calculating pipe-wall effects (Eqs. 92 and 93).	$Re$	Reynolds number (Eq. 65).
$g_1, g_2,$ $g_3, \dots$	Secondary (generator) coil sections (Fig. 3).	$RE400$	Reynolds number for a fluid temperature of 400°C (p. 26).
$G$	Circumferential conductance (Eq. 12).	$s$	Value of $s$ corresponding to the channel cross-section area (Eq. 61).
$G_p$	Gain factor for the flowmeter (Eq. 97).	$s400$	Selected value of $s$ for a fluid temperature of 400°C (p. 26).
$H$	Circumferential conductance of a length $L$ of pipe (Eq. 85).	$t$	Time (Eq. 3).
$H_0$	Axial component of field intensity (Eq. 15).	$t$	Thickness of the pipe (Eq. 85).
$H_d$	Axial component of field intensity at the axis of the flow channel (Eq. 27).	$u$	Velocity of a mutual-inductor coil (Eq. 7).
$H_{d0}$	Axial component of field intensity at the end of a primary coil section at radius $r$ (Eq. 36).	$u$	Velocity of a hollow-cylindrical element of fluid (Eq. 56).
$H_{d0}$	Axial component of field intensity at the axis of a primary coil section (Eq. 38).	$u_a$	Average fluid velocity (Eq. 58).
$H_{0g}$	Field at the axis due to the secondary coil current (Eq. 47).	$u_m$	Maximum fluid velocity (Eq. 65).
$H_{sd}$	Axial component of field intensity at the outside fluid surface at a primary-secondary coil interface (Eq. 44).	$V$	Secondary voltage developed by the motion of the fluid (Eq. 56).
$H_s$	Axial component of field intensity inside the pipe (Eq. 88).	$V_2$	Secondary voltage (Eq. 5).
$H_p$	Axial component of field intensity outside the pipe (Eq. 90).	$V_{2u}$	Motional component of voltage (Eq. 8).
$H_{sg}$	Axial component of field intensity at the outside fluid surface due to a current in the secondary coils (Eq. 99).	$V_n$	Normalized signal voltage (Eq. 95).
$I$	Circumferential current in the pipe (Eq. 89).	$VR, V_R$	Real and imaginary components of signal voltage (p. 23).
$I_1$	Current in the primary winding of a mutual inductor (Eq. 1).	$VS, VA$	Magnitude and phase angle of signal voltage (p. 23).
$I_2$	Current in the secondary winding of a mutual inductor (Eq. 1).	$x$	Distance between the coils of a mutual inductor (Eq. 1).
$I$	Circumferential current in the fluid inside a secondary coil (Eq. 47).	$x$	Dimensionless variable in the Bessel equation (proportional to a radius) (Eq. 22).
$I_d$	Current in the primary winding of a flowmeter (p. 10).	$x_1$	The value of the dimensionless variable corresponding to radius $r_1$ (Eq. 30).
$I_g$	Current in the secondary winding of a flowmeter (p. 10).	$x_5$	The value of the dimensionless variable corresponding to the radius of the flow channel (Eq. 46).
$I_{1m}$	Maximum value of $i_1$ (Eq. 3).	$\alpha_1$	Time-displacement angle (Eq. 3).
$j$	Square root of $-1$ (Eq. 13).	$\mu$	Permeability of the fluid (Eq. 18).
$K_{ihd}$	Dimensionless constant, depending on the geometry of the primary coil system (Eq. 45).	$\rho$	Resistivity of the fluid (Eq. 12).
$K_{ihg}$	Dimensionless constant, depending on the geometry of the secondary coil system (Eq. 48).	$\rho_f$	Resistivity of the fluid (same as $\rho$ , but used to distinguish it from $\rho_B$ ) (Eq. 87).
		$\rho_p$	Resistivity of the pipe (Eq. 85).
		$\phi$	Magnetic flux (Eq. 14).
		$\phi_1$	Flux within radius, $r_1$ , at the end of a primary coil (Eq. 36).
		$\omega$	$2\pi$ times the frequency of the primary current

NOTE: Numbers in parentheses indicate where symbols are first used.



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#### ABSTRACT

In the eddy-current flowmeter, motion of a fluid is measured through inductive coupling to exciting and sensing windings. An optimum excitation frequency, depending on the channel size and the fluid properties, yields maximum sensitivity. Operation in the region of the optimum frequency minimizes temperature errors and reduces velocity-profile effects.

#### INTRODUCTION

The eddy-current flowmeter is an ac-operated device in which eddy currents are induced in the (conducting) fluid by coupling to a primary coil system. Signal voltages are generated in a normally balanced secondary coil system through the distortion in the eddy-current pattern caused by the motion of the fluid.

A device using this principle was patented by Lehde and Lang.<sup>1</sup> An encapsulated coil system immersed in the flow stream is described in the patent. This configuration was used in the British Prototype Fast Reactor<sup>2</sup> as a flow-failure monitor. Calibration difficulties were experienced that did not rule out its use as a flow-failure monitor, however.

Shercliff<sup>3</sup> described the eddy-current flowmeter briefly and suggested an alternate construction with solenoidal coils wound around the outside of a cylindrical flow channel. Wiegand<sup>4</sup> presented the results of a preliminary analysis of the external-coil configuration. This report presents, in complete form, a refined version of the earlier report.

Results of an experimental program on the eddy-current flowmeter have been presented by Wiegand and Michels.<sup>5</sup>

Flowmeter types in a high-temperature application were compared by Popper, Wiegand, and Glass.<sup>6</sup>

## ELECTRODYNAMIC RECIPROCITY

The solution of many problems involving systems having relations between mechanical variables (velocity and force) and electromagnetic variables (voltage, current, magnetic induction, and magnetizing force) are simplified by use of the electrodynamic-reciprocity principle. This principle applies to systems having between two terminals a component of induced voltage that is proportional to the velocity of some member, and a force on the same member that is proportional to the current into and out of the terminals. With a consistent set of units, the ratio of voltage to velocity is equal to the ratio of force to current.

Electrodynamic reciprocity applies to a large number of fixed-field devices, such as dc motors and generators, electrodynamic speakers, and microphones. The principle does not apply directly to square-law force devices, such as lifting magnets and nonpolarized relays, which, in the absence of a coil current, have no terminal voltage associated with a velocity.

The principle applies to ac devices having the required linear relations between selected components of current, voltage, force, and velocity. Consider, for example, the variable mutual inductor of Fig. 1 in which currents in the coils cause a force between them.

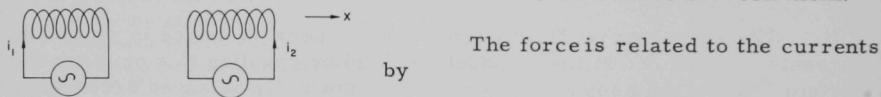


Fig. 1. Forces between Coils

$$F = i_1 i_2 \frac{dM}{dx}, \quad (1)$$

where  $F$ ,  $i_1$ , and  $i_2$  are, in general, time-varying quantities, or

$$\frac{F}{i_2} = i_1 \frac{dM}{dx}. \quad (2)$$

Let  $i_1$  have a sinusoidal variation,

$$i_1 = I_{1m} (\sin \omega t + \alpha_1). \quad (3)$$

Then

$$\frac{F}{i_2} = \frac{dM}{dx} I_{1m} \sin (\omega t + \alpha_1). \quad (4)$$

Now, replace the current source on the right-hand coil by a voltage indicator, as in Fig. 2.



Fig. 2

Motional Voltage

If the coil is moved in the  $x$  direction with a velocity  $u$ , the induced voltage is given by

$$V_z = M \frac{di_1}{dt} + i_1 \frac{dM}{dt} \quad (5)$$

$$= M \frac{di_1}{dt} + i_1 \frac{dM}{dx} \frac{dx}{dt} \quad (6)$$

$$= M \frac{di_1}{dt} + ui_1 \frac{dM}{dx}. \quad (7)$$

The component of voltage due to the motion is the right-hand term in Eq. 7. Also,

$$\frac{V_{zu}}{u} = i_1 \frac{dM}{dx}. \quad (8)$$

With the same sinusoidal variation on  $i$  as in the previous case, and with a constant or slowly varying velocity  $u$ ,

$$\frac{V_{zu}}{u} = \frac{dM}{dx} I_{1m} \sin(\omega t + \alpha_1). \quad (9)$$

By comparison with Eq. 4,

$$\frac{V_{zu}}{u} = \frac{F}{i_2}. \quad (10)$$

In converting to the complex form for the ac quantities, we should write Eq. 10 as

$$\frac{V_{zu}}{u} = \left[ \frac{F}{i_2} \right], \quad (11)$$

where the brackets indicate that, while the quotient is a complex number, it is not the quotient of two complex numbers.  $F$  consists of a double-frequency component superimposed on a constant value, and thus cannot be represented by a complex number. However, from Eq. 4 the quotient is a simple sinusoid at fundamental frequency and its complex notation is, therefore, legitimate.

Analyzing the eddy-current flowmeter on the basis of motor action and then applying the reciprocity principle lend a degree of insight into the analysis not possible were the voltage calculated directly.

### SIGNAL-PROFILE FUNCTION

In the eddy-current flowmeter, signal voltages are generated through the interaction of the axial fluid velocity with the radial component of the magnetic induction of the primary coil system. Radial fluxes are created more effectively by a group of short primary coil sections with suitable polarity reversals than by a single long coil. An example of this configuration is shown in Fig. 3, which is a section view of a flowmeter with four primary (driver) and four secondary (generator) coil sections wound around the flow channel. The central coil sections are wound in unit pairs (for example,  $d_2$  and  $d_3$  are wound as a single coil); division into sections as shown simplifies the analysis in that the signal voltage per primary-secondary section can be calculated.

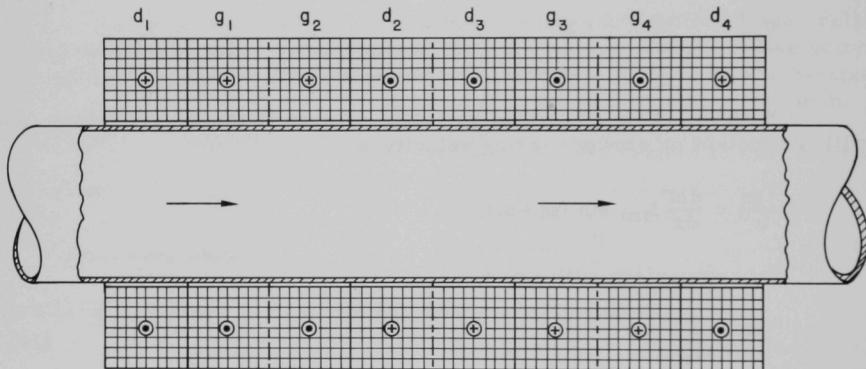


Fig. 3. Eddy-current Flowmeter with Four Primary-Secondary Sections

The system will be analyzed on the basis of the force that results on the fluid when ac currents  $I_g$  and  $I_d$  are made to flow in the secondary and primary coils, respectively. Application of the reciprocity principle will then give the secondary voltage that results from the fluid motion.

Although the coil sections of the proportions shown in Fig. 3 can hardly be considered "long," cylindrical geometry will be assumed in the calculation of the field-strength variation inside the coils. Under this assumption, a form of Bessel's equation results for which solutions are functions of radius alone. End-effect corrections may be applied later by

use of test data or by a more extensive analysis. The rationalized system of MKS (meter-kilogram-second) units will be used in the analysis.

The ac field-strength variation in a cylindrical conductor with axial excitation will be calculated first.

The circumferential fluid conductance of a hollow cylindrical element at radius  $r$  within a coil of length  $\ell$  is given by

$$dG = \frac{\ell dr}{2\pi r\rho}, \quad (12)$$

where  $\rho$  is the resistivity of the fluid. The current in the element is

$$di = j\omega\varphi dG, \quad (13)$$

where  $\varphi$  is the flux linking the element. Or, from Eq. 12,

$$di = j\omega\varphi \frac{\ell dr}{2\pi r\rho}. \quad (14)$$

But, by Ampere's law (in rationalized units),

$$di = \ell(H + dH) - \ell H = \ell dH, \quad (15)$$

where  $H$  is the axial component of field strength just inside the element, and  $dH$  is the change over its thickness. From Eqs. 14 and 15,

$$\varphi = \frac{2\pi\rho}{j\omega} r \frac{dH}{dr}. \quad (16)$$

Differentiating gives

$$\frac{d\varphi}{dr} = \frac{2\pi\rho}{j\omega} \left( r \frac{d^2H}{dr^2} + \frac{dH}{dr} \right). \quad (17)$$

But

$$d\varphi = 2\pi\mu H r dr, \quad (18)$$

where  $\mu$  is the permeability of the fluid, or

$$\frac{d\varphi}{dr} = 2\pi\mu H r. \quad (19)$$

So, from Eq. 17,

$$\frac{j\omega\mu}{\rho} Hr = r \frac{d^2H}{dr^2} + \frac{dH}{dr}, \quad (20)$$

or

$$r^2 \frac{d^2H}{dr^2} + r \frac{dH}{dr} - \frac{j\omega\mu Hr^2}{\rho} = 0. \quad (21)$$

One of the standard forms of the Bessel equation is obtained by substituting

$$\frac{\omega\mu r^2}{\rho} = x^2. \quad (22)$$

Then, Eq. 21 becomes

$$x^2 \frac{d^2H}{dx^2} + x \frac{dH}{dx} - jHx^2 = 0 \quad (23)$$

or

$$\frac{d^2H}{dx^2} + \frac{1}{x} \frac{dH}{dx} - jH = 0. \quad (24)$$

The general solution of Eq. 24 is

$$H = C_1[\text{ber}(x) + j \text{bei}(x)] + C_2[\text{ker}(x) + j \text{kei}(x)], \quad (25)$$

where  $\text{ber}(x)$ ,  $\text{bei}(x)$ ,  $\text{ker}(x)$ , and  $\text{kei}(x)$  are the Kelvin functions of zero order (from p. 379 of Ref. 7). At the axis of the conduit,

$$x = r = 0, \quad (26)$$

and

$$H = H_0. \quad (27)$$

From p. 431 of Ref. 7,

$$\text{ber}(0) = 1.0, \quad (28)$$

$$\text{bei}(0) = 0, \quad (29)$$

$$\text{ker}(0) = \infty, \quad (30)$$

and

$$\text{kei}(0) = -\pi/4. \quad (31)$$

Since  $H_0$ , the field at the axis, is finite,

$$C_2 = 0 \quad (32)$$

and

$$C_1 = H_0. \quad (33)$$

Therefore,

$$H = H_0[\text{ber}(x) + j \text{ bei}(x)], \quad (34)$$

or, if the complete complex variable is denoted by  $\text{be}(x)$ ,

$$H = H_0 \text{be}(x). \quad (35)$$

In line with the signal-per-section system of analysis, it will be assumed that the axial field component at the fluid surface is the same in magnitude at each primary-secondary juncture. With a limited number of sections, this assumption is not strictly true. For example, in Fig. 3, the surface field at the  $d_1g_1$  juncture is somewhat smaller than that at the  $d_2g_2$  juncture, since the  $d_3$  section adds somewhat to the field produced by  $d_2$ . This error will become smaller as the number of sections is increased, since it applies mainly to the end sections. Even with a limited number of sections, an average field value can be selected to minimize errors.

An important consequence of the above assumption is that all the axial flux issuing from the end of a primary section becomes the radial flux linking the fluid within a secondary section. Calculation of the radial flux in the fluid inside a secondary coil then becomes a simple matter.

The flux within a radius  $r_1$  at the end of a primary coil is given by

$$\varphi_1 = \int_0^{r_1} 2\pi\mu r H_d dr, \quad (36)$$

where  $H_d$  is the axial field intensity at the end of a primary coil at radius  $r$ .

From Eq. 22,

$$\varphi = \frac{2\pi\rho}{\omega} \int_0^{x_1} H_d x dx, \quad (37)$$

where

$$x_1 = \sqrt{\frac{\omega\mu}{\rho}} r_1$$

and, from Eq. 34,

$$\varphi = \frac{2\pi\rho}{\omega} H_{0d} \left[ \int_0^{x_1} \text{ber}(x)x \, dx + j \int_0^{x_1} \text{bei}(x)x \, dx \right], \quad (38)$$

where  $H_{0d}$  is the field at the axis of the primary coil. From p. 380 of Ref. 7,

$$\int \text{ber}(x)x \, dx = x \frac{d}{dx} \text{bei}(x) \quad (39)$$

and

$$\int \text{bei}(x)x \, dx = -x \frac{d}{dx} \text{ber}(x). \quad (40)$$

Thus, dropping the 1 subscripts of  $x$  in Eq. 38 after applying the limits of integration results in

$$\varphi = \frac{2\pi\rho H_{0d}}{\omega} x \left[ \frac{d}{dx} \text{bei}(x) - j \frac{d}{dx} \text{ber}(x) \right],$$

or

$$\varphi = \frac{2\pi\rho H_{0d}}{\omega} x(-j) \frac{d}{dx} \text{be}(x). \quad (41)$$

The average radial component of magnetic induction inside a secondary coil at radius  $r$  is

$$B_r = \frac{\varphi}{2\pi r l_g}, \quad (42)$$

where  $l_g$  is the length of one secondary coil section. Using the variable transformation, Eqs. 22 and 41, produces

$$B_r = \sqrt{\frac{\mu_0}{\omega}} \frac{H_{0d}}{l_g} (-j) \frac{d}{dx} \text{be}(x). \quad (43)$$

The field at the axis can be related to that at the outside surface by use of Eq. 35. Thus,

$$B_r = \sqrt{\frac{\mu_0}{\omega}} \frac{H_{sd}}{l_g} (-j) \frac{\frac{d}{dx} \text{be}(x)}{\text{be}(x_s)}, \quad (44)$$

where  $H_{sd}$  is the field strength at the outside fluid surface at the primary-secondary interface and is given by

$$H_{sd} = K_{ihd} \frac{N_d I_d}{D_c}, \quad (45)$$

where  $N_d$  is the number of turns in a primary section,  $D_c$  is the channel diameter,  $I_d$  is the primary current, and  $K_{ihd}$  is a real, dimensionless constant depending on the system geometry. Thus, in terms of the primary current,

$$B_r = \sqrt{\frac{\mu_0}{\omega}} \frac{K_{ihd} N_d I_d}{D_c \ell_g} (-j) \frac{\frac{d}{dx} be(x)}{be(x_s)}. \quad (46)$$

If a current of the same frequency as that of the primary current is made to flow in the secondary coils, the resulting induced currents in the fluid react with the radial induction of Eq. 46 to produce an axial force on the fluid. Use of these hypothetical currents and forces with the reciprocity principle yields the sensitivity factor of the flowmeter as actually used.

The current in a hollow cylindrical element of the fluid within a secondary coil is, from Eqs. 15 and 35,

$$dI = \ell_g H_{0g} \frac{d}{dx} be(x) dx, \quad (47)$$

where  $\ell_g$  is the length of a secondary coil and  $H_{0g}$  is the field at the axis due to the secondary coil current.

Using the relations between secondary surface field and secondary coil current, and between the axial and surface fields, as was done for the primary, produces

$$dI = \frac{N_g I_g K_{ihg} \ell_g}{D_c} \frac{\frac{d}{dx} be(x) dx}{be(x_s)}, \quad (48)$$

or

$$\frac{dI}{I_g} = \frac{N_g K_{ihg} \ell_g}{D_c} \frac{\frac{d}{dx} be(x) dx}{be(x_s)}. \quad (49)$$

The circumference of the element, using the variable transformation Eq. 22, is

$$2\pi r = 2\pi x \sqrt{\frac{\rho}{\omega\mu}}. \quad (50)$$

The product of Eqs. 46, 49, and 50 gives the force per primary current. Thus,

$$\left[ \frac{dF}{I_g} \right] = \frac{2\pi\rho}{\omega} \frac{K_{ihd} K_{ihg} N_d N_g I_d}{D_c^2} (-j) \frac{\left[ \frac{d}{dx} b e(x) \right]^2 x dx}{[b e(x_s)]^2}. \quad (51)$$

From Eq. 22

$$x^2 = \frac{2\pi f \mu r^2}{\rho} = \frac{2f\mu A}{\rho}, \quad (52)$$

where  $A$  is the area enclosed by the element. So,

$$x dx = \frac{f\mu}{\rho} dA \quad (53)$$

where  $dA$  is the area of the element. The area,  $A_c$ , of the channel is related to its diameter,  $D_c$ , by

$$D_c^2 = \frac{4}{\pi} A_c. \quad (54)$$

Thus, applying Eqs. 53 and 54 to Eq. 51 produces

$$\left[ \frac{dF}{I_g} \right] = \frac{\pi}{4} \mu K_{ihd} K_{ihg} N_d N_g I_d (-j) \frac{\left[ \frac{d}{dx} b e(x) \right]^2}{[b e(x_s)]^2} \frac{dA}{A_c}. \quad (55)$$

Using reciprocity and multiplying by  $N_{dg}$ , the number of primary-secondary sections, to obtain the total signal voltage generated by a cylindrical element of fluid, we obtain

$$dV = \frac{\pi}{4} \mu K_{ihd} K_{ihg} N_d N_g N_{dg} I_d (-j) \frac{\left[ \frac{d}{dx} b e(x) \right]^2}{[b e(x_s)]^2} \frac{dA}{A_c} u, \quad (56)$$

where  $u$  is the fluid velocity in the element.

The signal-profile function is defined as

$$F_{sp}(x, x_s) = (-j) \frac{\left[ \frac{d}{dx} b e(x) \right]^2}{[b e(x_s)]^2}. \quad (57)$$

Thus,

$$V = \frac{\pi}{4} \mu K_{ihd} K_{ihg} N_d N_g N_{dg} I_d u_a \sum_{A=0}^{A=A_c} F_{sp} \frac{u}{u_a} \frac{\Delta A}{A_c}, \quad (58)$$

where  $u_a$  is the average velocity in the channel.

The factors to the left of the summation sign in Eq. 58 are combined as a normalization factor.

Thus,

$$K_n = \frac{\pi}{4} \mu K_{ihd} K_{ihg} N_d N_g N_{dg} I_d u_a, \quad (59)$$

and

$$V = K_n \sum_{A=0}^{A=A_c} F_{sp} \frac{u}{u_a} \frac{\Delta A}{A_c}. \quad (60)$$

In the computer program for  $F_{sp}$  (program SIGPROF in Appendix A) and in related programs, new variables relating directly to the first power of areas, frequency, fluid permeability, and fluid resistivity, are introduced.

Thus,

$$s = \frac{x_s^2}{2} \quad (61)$$

and

$$a = \frac{x^2}{2}, \quad (62)$$

and, from Eq. 52,

$$s = \frac{f \mu A_c}{\rho} \quad (63)$$

and

$$a = \frac{f \mu A}{\rho}. \quad (64)$$

In Figs. 4 and 5, the in-phase and quadrature components of the signal-profile function are plotted against relative radius squared. Zero corresponds to the channel axis, and unity to its outside surface.

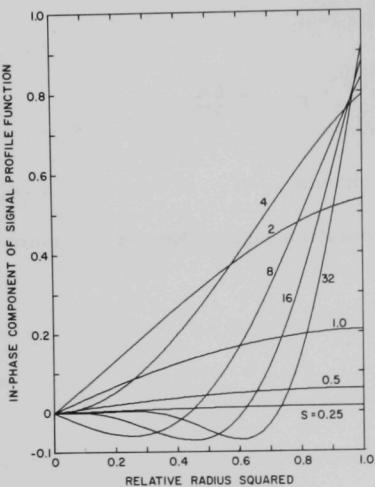


Fig. 4. In-phase Component of Signal-profile Function

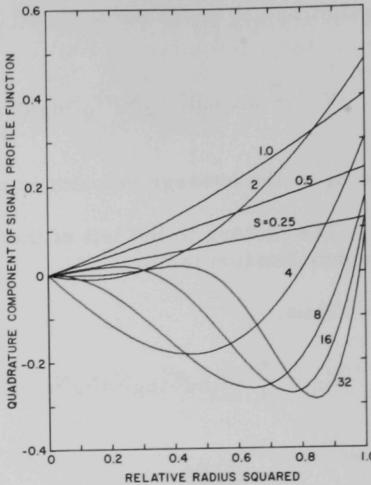


Fig. 5. Quadrature Component of Signal-profile Function

These curves, with Eq. 58, can be used to calculate the voltage for any axisymmetric velocity profile in the fluid. Signal components in phase and in quadrature with the primary current are given. From Eq. 63, the curves at various  $s$  values represent different operating frequencies for given channel size and fluid properties. The field constants,  $K_{ihd}$  and  $K_{ihg}$  in Eqs. 58 and 59, can be calculated by the method of Brown and Flax.<sup>8</sup>

The curves of Figs. 4 and 5 give information as to the type of read-out equipment required with the eddy-current flowmeter and as to desirable operating frequencies. The signal amplifier could be made to read (a) the total signal, (b) the signal component in phase with the primary current, or (c) the quadrature component, where the possible measuring systems are listed in the order of increasing complexity.

The rapidly rising curves in the region of the channel surface in Fig. 5 for the quadrature signal, coupled with the rapidly changing fluid velocity (for typical profiles) in this region, make for calibration instability due to minor changes in the surface condition of the pipe wall. Also, with portions of the curves above and below the zero line, it is easy to imagine velocity profiles that would give gross measurement errors, such as a false indication of zero flow, or even a phase reversal, yielding a false indication of reversed flow direction.

These facts, and the generally higher signal levels indicated in Fig. 4, make the selection of the in-phase component preferable. The

concave-downward nature of the in-phase curves for  $s$  values less than 4 reduces the sensitivity to conditions of the pipe wall, and tends to give a better measurement of total flow, in spite of a changing velocity profile.

In Fig. 5, a net positive area is shown for curves for  $s = 2$  and lower, and net negative areas for  $s = 4$  and higher. We can expect that an  $s$  value exists between 2 and 4 for which the net area is zero. If an operating frequency is chosen corresponding to this  $s$  value, the quadrature signal (for uniform fluid velocity) is zero and the total signal is in phase with the primary current.

It is also evident from Fig. 4 that the in-phase component has a maximum area for  $s$  between 2 and 4. The desirability of choosing the operating frequency providing an in-phase signal is quite clear. Not only is the signal a maximum here, but the signal-conditioning equipment is greatly simplified by the elimination of the quadrature component. In particular, operation of a phase-sensitive detector with a primary-current reference is more reliable in the absence of the quadrature signal.

At this point, these conclusions hold, strictly, for uniform fluid velocity only. However, a condition of near-zero phase shift and signal maximum holds for a wide range of velocity profiles at a particular  $s$  value or operating frequency.

A degree of nonlinearity is introduced into the flowmeter response by the combined effects of a radius-sensitive signal (Figs. 4 and 5) and a velocity profile that changes with the average flow velocity. In the next section, velocity profiles are developed for a range of Reynolds numbers for the fluid. The velocity-profile data are then combined with the signal-profile relations to give the flowmeter response as a function of volume rate and operating frequency.

### VELOCITY PROFILES

Flowmeters in liquid-metal reactor systems typically operate with the fluid in the turbulent condition. Various relations (some with analytic foundations) have been used to characterize the velocity profiles at various Reynolds numbers. A simple power law, empirically adjusted to fit test data, is sufficient for our purpose and is convenient to use.

Knudsen and Katz (p. 149 of Ref. 9) plotted a wide range of data from various investigators showing the variation of the ratio of average to maximum velocity with Reynolds numbers on semilogarithmic scales. In the turbulent region, the data points fit a straight line very closely. That is,

$$\frac{u_a}{u_m} = A + B \log Re, \quad (65)$$

where  $u_a$  and  $u_m$  are the average and maximum velocities. The values

$$A = 0.655 \quad (66)$$

and

$$B = 0.035 \quad (67)$$

fit the straight part of the curve. Thus,

$$\frac{u_a}{u_m} = 0.655 + 0.035 \log Re. \quad (68)$$

When an empirical relation is chosen for the fluid velocity at intermediate radii, condition 68 must be fulfilled, and, for continuity at the axis, the slope of the velocity-radius curve should be zero at zero radius. The following relation meets the requirement for zero slope at the axis and can be made to meet condition 68 through the proper choice of the exponent  $n$ :

$$u = u_m \left[ 1 - \left( \frac{r}{R} \right)^n \right], \quad (69)$$

where  $R$  is the channel radius, and  $u$  is the velocity at radius  $r$ .

The area integral of the velocity must equal the average velocity times the channel area. That is,

$$\int u dA = A_c u_a, \quad (70)$$

where  $A_c$  is the channel area.

In terms of the radius and our assumed velocity relation,

$$\int_0^R u_m \left[ 1 - \left( \frac{r}{R} \right)^n \right] 2\pi r dr = \pi R^2 u_a, \quad (71)$$

or

$$\int_0^R \left[ 1 - \left( \frac{r}{R} \right)^n \right] r dr = \frac{R^2 u_a}{2u_m}. \quad (72)$$

Integration of Eq. 72 yields

$$n = \frac{2 \frac{u_a}{u_m}}{1 - \frac{u_a}{u_m}}. \quad (73)$$

A normalized form of Eq. 69 is

$$\frac{u}{u_a} = \frac{1}{\frac{u_a}{u_m}} \left[ 1 - \left( \frac{r}{R} \right)^n \right], \quad (74)$$

or, in terms of the relative area,

$$\frac{u}{u_a} = \frac{1}{\frac{u_a}{u_m}} \left[ 1 - \left( \frac{A}{A_c} \right)^m \right], \quad (75)$$

where  $A_c$  is the channel area and  $A$  is the partial area. That is,

$$A = \pi r^2, \quad (76)$$

$$A_c = \pi R^2, \quad (77)$$

and

$$m = \frac{n}{2} = \frac{\frac{u_a}{u_m}}{1 - \frac{u_a}{u_m}}. \quad (78)$$

From Eqs. 63 and 64, Eq. 75 may be written in terms of the previously defined variables as follows:

$$\frac{u}{u_a} = \frac{1}{\frac{u_a}{u_m}} \left[ 1 - \left( \frac{a}{s} \right)^m \right]. \quad (79)$$

Since  $u_a/u_m$  is given by Eq. 68, the velocity profile for the turbulent flow condition is determined.

At low volume rates, the flow may become laminar, in which case the velocity profile becomes parabolic (p. 86 of Ref. 9).

In our notation,

$$u = 2u_a \left[ 1 - \left( \frac{r}{R} \right)^2 \right] = 2u_a \left( 1 - \frac{a}{s} \right), \quad (80)$$

or

$$\frac{u}{u_a} = 2 \left( 1 - \frac{a}{s} \right). \quad (81)$$

By a comparison with Eq. 79, as the fluid drops into the laminar condition, the exponent  $m$  takes on the constant value 1, and  $u_a/u_m$  becomes  $1/2$ .

Table I shows the variations of the parameters describing the velocity profile.

TABLE I. Velocity-profile Variables

Re	Flow Condition	$\frac{u_a}{u_m}$	m	Re	Flow Condition	$\frac{u_a}{u_m}$	m
$<4 \times 10^3$	Laminar	0.500	1.00	$1 \times 10^6$	Turbulent	0.865	6.41
$4 \times 10^3$	Turbulent	0.781	3.57	$2 \times 10^6$	Turbulent	0.876	7.03
$1 \times 10^4$	Turbulent	0.795	3.88	$4 \times 10^6$	Turbulent	0.861	7.78
$2 \times 10^4$	Turbulent	0.806	4.14	$1 \times 10^7$	Turbulent	0.900	9.00
$4 \times 10^4$	Turbulent	0.816	4.37	$2 \times 10^7$	Turbulent	0.911	10.18
$1 \times 10^5$	Turbulent	0.830	4.88	$4 \times 10^7$	Turbulent	0.921	11.67
$2 \times 10^5$	Turbulent	0.841	5.27	$1 \times 10^8$	Turbulent	0.935	14.38
$4 \times 10^5$	Turbulent	0.851	5.71	$\infty$	Uniform	1.000	$\infty$

Figures 6 and 7 show velocity profiles on the basis of radius and area, respectively.

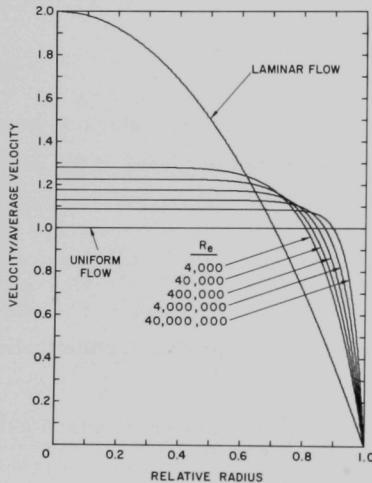


Fig. 6. Velocity Profiles, Radius Base

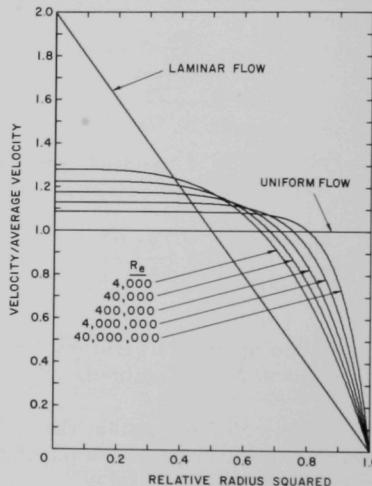


Fig. 7. Velocity Profiles, Area Base

#### VARIATION OF SIGNAL WITH REYNOLDS NUMBER AND FREQUENCY

In program FLOMGEN (Appendix B), Eqs. 57, 68, 78, 79, and 81 are used to perform the summation indicated in Eq. 60. Separate program branches are used for the laminar, turbulent, and uniform flow conditions.

In calculating the signal-profile function, we use the following expansion:

$$-j \left[ \frac{d}{dx} \text{be}(x) \right]^2 = 2d\text{ber}(x)d\text{bei}(x) + j[d\text{bei}^2(x) - d\text{ber}^2(x)], \quad (82)$$

where

$$d\text{ber}(x) = \frac{d}{dx} \text{ber}(x), \quad (83)$$

and

$$d\text{bei}(x) = \frac{d}{dx} \text{bei}(x). \quad (84)$$

The subroutines for all programs are shown separately in Appendix E. Those for the Bessel functions and their derivatives use polynomial approximations of eight terms, yielding an accuracy to about 1 in  $10^8$ .

The programs were written to be compatible with the CDC 160A and CDC 3600 computers. Sums of 25 increments were used on the 160A, and 250 increments on the 3600. The small difference shown where the same program was run on both computers indicates that the 250-term sum is accurate to a fraction of 1%.

Complex variables were manipulated through the TORECT and TOPOLR subroutines, which convert from the polar to the rectangular forms and vice versa. Additions were performed in the rectangular form, and multiplications in the polar form.

Program FLOMGEN gives solutions of the normalized voltage function,  $V_n$ , in rectangular and polar forms.  $V_n$  is the summation term in Eq. 60. In the program, VR, VI, VS, and VA indicate the real component, imaginary component, magnitude, and phase angle of the function, respectively. The primary current,  $I_d$ , is the reference.

Values of  $s$  in geometric steps of  $2^{1/6}$  covering the range 0.25 through 32.0 are shown in the output of the program. For each  $s$ , solutions are given for laminar flow, uniform flow, and turbulent flow for Reynolds numbers from 4000 (the approximate lower limit for turbulent flow) through  $10^7$ . The 4, 10, 20, 40 system of increase is used here. The normalized voltage function is plotted in Figs. 8-11.

The phase angle of the signal, VA, for all flow conditions goes through zero within the range of  $s$  values, 2.51984 to 3.17480. Also, the in-phase signal, VR, and the signal magnitude, VS, have maximums in this  $s$  range.

Other advantages, more important than mere high signal levels, result if operating conditions are selected to give an  $s$  value near a signal maximum. These advantages are apparent from a study of the  $s$  variable, as defined in Eq. 63.

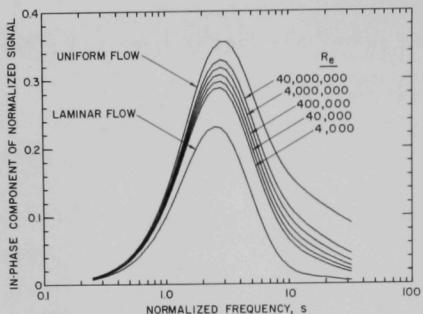


Fig. 8. In-phase Component of Normalized Signal

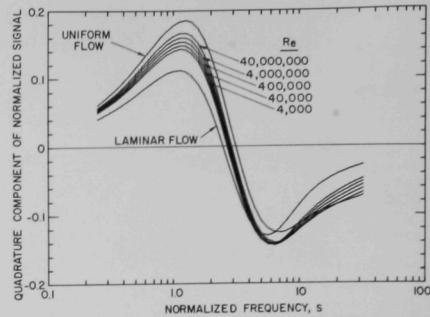


Fig. 9. Quadrature Component of Normalized Signal

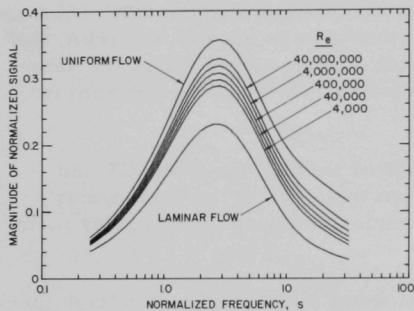


Fig. 10. Magnitude of Normalized Signal

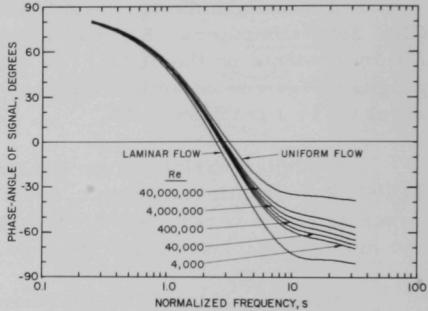


Fig. 11. Phase Angle of Signal

With  $s$  in the region of a signal maximum, calibration changes due to unknown variations in any of the parameters making up the  $s$  variable are minimized. For example, the requirement on the stability of the carrier frequency supply are made less stringent than when operating with  $s$  in an off-maximum region, where small variations in  $s$  cause much greater variations in signal level. This philosophy applies particularly to the matter of temperature errors, which are investigated in the next section.

Certain disadvantages of using the quadrature signal are apparent from Fig. 9. Either the signal maximum, in the region slightly above  $s = 1.0$ , or the signal minimum, near  $s = 7.0$ , could be selected as operating points. The smaller spread of the curves about the negative peak makes this operating condition the preferable one. In either case, broad swings in fluid resistivity under transient temperature conditions could move the operating point along the steep curves between the positive and negative peaks and give a false indication of a flow reversal.

This fact, the higher signal levels shown for the in-phase or total signals, and the greater complexity of the equipment that would be required for selecting the quadrature signal component, make the use of the in-phase or total signal preferable.

As mentioned previously, zero-phase, and maximum in-phase and total signals occur for  $s$  values in the range  $2.51984 \leq s \leq 3.17480$ . The lower and higher limits represent the zero-phase condition for the laminar and uniform flow conditions, respectively.

Table II shows the range of operating frequencies giving the higher  $s$  value for a range of channel diameters, for sodium at  $400^{\circ}\text{C}$  ( $\rho = 0.2193 \times 10^{-6}$  ohm-meter).

TABLE II. Optimum Operating Frequencies for Sodium at  $400^{\circ}\text{C}$

Channel Diameter		Frequency, Hz	Channel Diameter		Frequency, Hz
Inches	Meters		Inches	Meters	
0.32	0.0081	10680	4.0	0.1016	68.34
0.40	0.0102	6834	5.0	0.1270	43.74
0.50	0.0127	4374	6.4	0.1626	26.70
0.64	0.0163	2670	8.0	0.2032	17.09
0.80	0.0203	1709	10.0	0.2540	10.94
1.00	0.0254	1094	12.5	0.3175	7.000
1.25	0.0318	700.0	16	0.4064	4.271
1.6	0.0406	427.1	20	0.5080	2.734
2.0	0.0508	273.4	25	0.6350	1.750
2.5	0.0635	175.0	32	0.8128	1.068
3.2	0.0813	106.8			

The low operating frequencies at the larger channel diameters may lead to serious response-time limitations. In these cases, there may be an advantage in using the quadrature signal at the negative response peak (Fig. 9), where the operating frequencies are about 2.2 times those shown in Table II. The greater complexity of temperature-compensation and quadrature-signal selecting equipment may be justified with flowmeters for large-diameter channels.

Another method of avoiding the extremely low frequencies at the large diameters is to use a series of encapsulated sensors of the type described by Lehde and Lang<sup>1</sup> at different radii in the flow channel, and to average the signals. The operating frequency should be as low as possible, consistent with response time requirements and signal levels. Although Figs. 4 and 5 hold, strictly, only for the construction using the internal field

of a coil system external to the flow channel, the extreme sensitivity to conditions very near the flow boundary at high operating frequencies will hold for the stream-immersed coil system as well.

### TEMPERATURE ERRORS

Temperature errors in the eddy-current flowmeter occur in two ways: (1) A change in resistivity of the fluid changes the  $s$  value for a given operating frequency; (2) changes in the viscosity and density of the fluid change the Reynolds number for a given volume rate and thereby change the velocity distribution of the fluid and, consequently, the generated signal level. The second effect is usually quite small, but is included in the calculations (see program FLOMTEMP, Appendix C).

A reference temperature is selected ( $400^{\circ}\text{C}$  in program FLOMTEMP) near the middle of an expected operating temperature range. A desirable  $s$  for fluid conditions at the reference temperature is chosen ( $s_{400} = 2 \times 2^{2/3}$  in program FLOMTEMP). In effect, this selection of  $s$  sets the operating frequency.

To provide for a range of normalized volume rates, a range of Reynolds numbers for fluid conditions at the reference temperature (RE400 in FLOMTEMP) is selected.

The fluid properties (resistivity, density, and viscosity) for a selected set of fluid temperatures have been calculated.<sup>10</sup> These values are used to determine the  $s$  value (for fixed frequency) and the Reynolds number (for the flow held at the particular RE400 value). The normalized signal levels are then calculated for each selected temperature. The complex components VR, VI, VS, and VA show the variation of the in-phase and out-of-phase components and the magnitude and phase of the normalized signal.

The values for RE400 = 4000 are unreliable for temperatures below the reference temperature ( $400^{\circ}\text{C}$ ) since the actual Reynolds number drops significantly below the limiting value for turbulence in this region.

The temperature effects for various flow conditions are shown in Figs. 12-15. The rising curves in Fig. 13 ranging from negative to positive signal values for all flow conditions indicate an essentially uncompensatable temperature effect for the quadrature signal at the chosen value of  $s_{400}$ . Of course, the  $s_{400}$  value was deliberately chosen to make the quadrature signal zero in the midtemperature range. Figure 13, therefore, presents a very pessimistic picture of the quadrature-signal performance. These curves are included only for completeness.

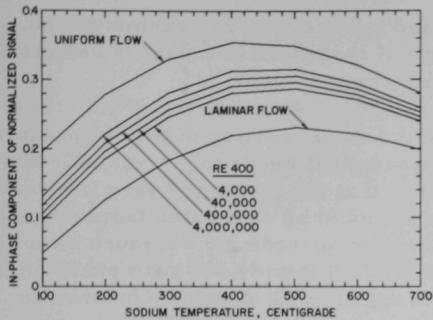


Fig. 12. Temperature Variation of In-phase Component

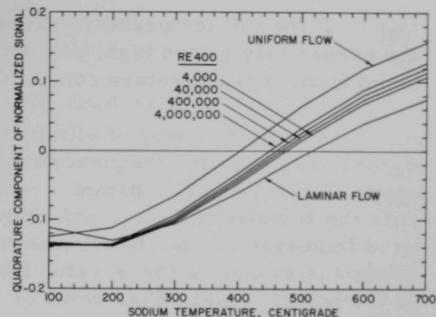


Fig. 13. Temperature Variation of Quadrature Component

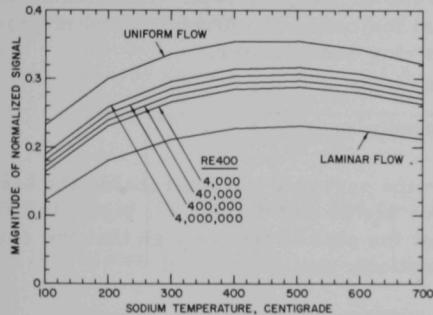


Fig. 14. Temperature Variation of Signal Magnitude

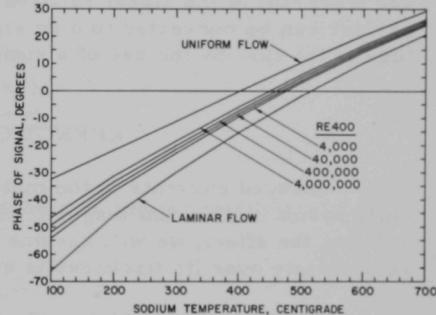


Fig. 15. Temperature Variation of Signal Phase

Table III summarizes the effects of temperature on the in-phase and total signals. These data are for constant-frequency operation with no form of temperature compensation.

TABLE III. Uncompensated Temperature Errors

In-phase Signal

Flow condition	Laminar	RE400 = 10,000	Uniform
Maximum temperature errors, %	±5.2	±9.2	±11.7
Temperature range, °C	340-700	287-700	202-700

Total Signal

Flow condition	Laminar	RE400 = 10,000	Uniform
Maximum temperature errors, %	±3.6	±4.6	±4.9
Temperature range, °C	289-700	288-700	258-700

If the full temperature ranges indicated are to be accommodated, the errors may be too high, particularly if the in-phase signal is selected. Some form of temperature compensation may be required.

An effective way of eliminating the major portion of the temperature errors is to vary the frequency in a manner that keeps the  $s$  variable constant. From Eq. 63, constant  $s$  requires that the frequency vary inversely with the resistivity. The fluid resistivity could be calculated from a measured temperature and the frequency adjusted accordingly. A much simpler method is to choose the  $s$  value (about 2.82) that provides zero phase angle in the output signal. This value of  $s$  is maintained, despite temperature variations, merely by adjusting the frequency to keep the output signal in phase with the exciting current. With this mode of operation, sensitivity is near maximum, and undesirable effects of quadrature-signal components are eliminated. For example, a flow reversal is indicated by a complete  $180^\circ$  reversal of the signal relative to the exciting current. The modulated carrier can be converted to a dc signal indicating the direction and magnitude of the flow by the use of a simple ring demodulator.

#### EFFECT OF PIPE WALL

Induced currents in the metal pipe carrying the fluid inside the test coils result in different magnetic fields inside and outside the pipe. In calculating the effect, we will assume that the pipe is thin enough that the current density over its thickness is essentially uniform.

The circumferential conductance of a length  $\ell$  of the pipe is

$$G_p = \frac{\ell t}{2\pi\rho_p \left( R + \frac{t}{2} \right)}, \quad (85)$$

where  $t$  is the pipe thickness,  $\rho_p$  its resistivity, and  $R$  is the inside radius, or channel radius, as before.

Uniform current density, as assumed, means that the flux in the pipe is negligible, and

$$i = j\omega\varphi G_p, \quad (86)$$

where  $\varphi$  is the flux in the fluid, as before.

The total flux is obtained by replacing  $x$  in Eq. 41 by  $x_s$  and  $\rho$  by  $\rho_f$  to distinguish from pipe resistivity  $\rho_p$ . The result is

$$\varphi = \frac{2\pi\rho_f H_0 d}{\omega} x_s(-j) \frac{d}{dx} \text{be}(x_s), \quad (87)$$

or, from Eq. 35, in terms of the field at the fluid surface

$$\varphi = \frac{2\pi\rho_f H_s}{\omega} x_s(-j) \frac{\frac{d}{dx} \text{be}(x_s)}{\text{be}(x_s)}. \quad (88)$$

Combining Eqs. 85, 86, and 88 produces

$$i = \frac{\rho_f H_s \ell t}{\rho_p \left( R + \frac{t}{2} \right)} \frac{x_s \frac{d}{dx} \text{be}(x_s)}{\text{be}(x_s)}, \quad (89)$$

or, by Ampere's law,

$$(H_p - H_s) \ell = \frac{\rho_f}{\rho_p} \frac{H_s \ell t}{R + \frac{t}{2}} \frac{x_s \frac{d}{dx} \text{be}(x_s)}{\text{be}(x_s)}, \quad (90)$$

where  $H_p$  is the field due to the coil just outside the pipe wall.

From Eq. 90,

$$\frac{H_p}{H_s} = 1 + \frac{t}{R + \frac{t}{2}} \frac{\rho_f}{\rho_p} \frac{x_s \frac{d}{dx} \text{be}(x_s)}{\text{be}(x_s)}. \quad (91)$$

Since the function of  $x_s$  at the right in Eq. 91 is a complex variable, the field is subject to a phase shift as well as a magnitude change through the pipe thickness.

With currents in both primary (driver) and secondary (generator) coils, the field change applies to both sets of coils. Thus, by reciprocity, the reduction in signal is the square of the reciprocal of  $H_p/H_s$  in Eq. 91.

In program PIPELOSS (Appendix D),

$$\text{FUNCR} = \text{real component of } \frac{x_s \frac{d}{dx} \text{be}(x_s)}{\text{be}(x_s)}, \quad (92)$$

and

$$\text{FUNCI} = \text{imaginary component of same.} \quad (93)$$

Values are computed for the same range of  $s$  values as in program FLOMGEN. The variables  $x_s$  and  $s$  are related by Eq. 61.

FUNCR and FUNCI from the computer program along with the pipe dimensions and the resistivities of pipe and fluid in Eq. 91 give the reciprocal of the field attenuation. That is,

$$\frac{H_p}{H_s} = 1 + \frac{t}{R + \frac{t}{2}} \frac{\rho_f}{\rho_p} [\text{FUNCR} + j \text{FUNCI}]. \quad (94)$$

For an example of a pipe-loss calculation, consider sodium at 400°C flowing in a stainless steel pipe with an inside diameter of 1.00 in. and a 0.0625-in. wall. Here,  $R = 0.500$  in.,  $t = 0.0625$  in.,  $\rho_f = 2.19 \times 10^{-7}$  ohm-meter, and  $\rho_p = 7.20 \times 10^{-7}$  ohm-meter.

$$\text{Take } s = 3.17. \text{ Then, from Table II, } f = 1094 \text{ Hz. From Eq. 61, } x_s = \sqrt{2 \times 3.17} = 2.52.$$

From the output of program PIPELOSS in Appendix D,  $\text{FUNCR} = 1.20$ , and  $\text{FUNCI} = 1.96$ . From Eq. 94,

$$\begin{aligned} \frac{H_p}{H_s} &= 1 + \frac{0.0625}{0.500 + 0.0625} \frac{2.19 \times 10^{-7}}{7.2 \times 10^{-7}} (1.20 + j 1.96) \\ &= 1.04 + j 0.0663 \\ &= 1.04 \text{ cjs } 3.65^\circ. \end{aligned}$$

Therefore,

$$\frac{H_s}{H_p} = 0.960 \text{ cjs}(-3.65^\circ).$$

Thus the magnetic field suffers an attenuation of 0.28 dB and a phase lag of 3.65°.

The effect occurs twice for the generated signal. Thus, the pipe causes a loss of 0.56 dB in magnitude and 7.30° in phase.

## DISCUSSION

In Eq. 60, the summation term, calculated in programs FLOMGEN and FLOMTEMP, is a dimensionless quantity, which is a function of the variable  $s$  and the velocity profile. A particular value of  $s$  (approximately 3) yields a maximum value of the function and eliminates its imaginary

component. From Eq. 63, the operating frequency must be made to vary inversely with the channel area to hold this fixed value of  $s$  in a line of flowmeters of varying sizes.

The actual signal voltage is the product of the dimensionless variable and  $K_n$ , defined by Eq. 59. Equation 60, showing this relation may be written

$$V = K_n V_n, \quad (95)$$

where  $V_n$  is the normalized voltage calculated in the various computer programs.

Since Eq. 59 contains none of the dimensions of the flowmeter, and since  $K_{ihd}$  and  $K_{ihg}$  are dimensionless quantities depending only on the proportions of the coil structure, it is evident that the flowmeter is basically a velocity-sensitive device. Except for the fact that increased primary ampere turns are possible in a larger structure, the signal levels should be the same in large and small flowmeters, if the fluid velocity is the same.

Furthermore, if the frequency is adjusted to provide the desirable value of  $s$ , the signal level becomes independent of the fluid resistivity. This leads us to expect that the flowmeter is operable with high-resistivity fluids as well as with the liquid metals. However, the maintenance of balance in the coil system would become increasingly difficult at the higher frequencies, since the false signal due to a residual mutual inductance between primary and secondary coils would be directly proportional to the frequency. This consideration places a practical limit on the fluid resistivity.

The performance tables and curves show about the same change in sensitivity between the laminar-flow condition and marginal turbulence, as over the entire turbulent range, approaching uniform flow. While a calibration factor independent of flow would be possible in the laminar case, the high density and low viscosity of typical liquid-metal coolants would force an extremely low fluid velocity with a resulting low sensitivity and over-size structure.

It is preferable to operate with the fluid in the turbulent state, avoiding the erratic transition to laminar flow by tapering down the flow channel to guarantee a Reynolds number of at least 4000 at the lowest calibrated volume rate. The minimum channel size is limited by the allowable pressure loss in the flowmeter. We have estimated that flow ranges of 60 dB or greater can be accommodated in liquid sodium with a net pressure loss limited to about 10 psi.

The spread of the turbulent curves indicates a nonlinear calibration, rather than an error. The effect can be compensated by a nonlinear meter scale or by a corrective nonlinearity in the amplifying equipment.

A gain factor, useful in flowmeter design calculations, can be obtained from Eqs. 59 and 95 by expressing  $u_a$  in terms of the volume rate and channel diameter, as follows:

$$u_a = \frac{q}{\frac{\pi}{4} D_c^2}, \quad (96)$$

where  $q$  is the volume rate, and  $D_c$  is the channel diameter. Thus,

$$G = \frac{V}{I_d q} = \frac{\mu K_{ihd} K_{ihg} N_d N_g N_{dg} V_n}{D_c^2}, \quad (97)$$

where  $G$  is the gain factor.  $K_{ihd}$  is defined by Eq. 45; thus,

$$K_{ihd} = \frac{D_c H_{sd}}{N_d I_d}. \quad (98)$$

The corresponding relation for  $K_{ihg}$  is

$$K_{ihg} = \frac{D_c H_{sg}}{N_g I_g}. \quad (99)$$

Note that  $H_{sd}$  is the surface field at the generator-driver interfaces, while  $H_{sg}$  is the average over the lengths of the generator coils.

The method of Brown and Flax<sup>8</sup> was used to calculate the field constants for the coil configuration of Fig. 3 (square coil section with inside radius equal to section side).

The average of the values at the four d-g interfaces yields

$$K_{ihd} = 0.646, \quad (100)$$

and the average value over the lengths of the two generator double sections is

$$K_{ihg} = 1.06. \quad (101)$$

Putting these values,  $\mu = 4\pi \times 10^{-7}$ , and  $V_n = 0.31$  (a representative peak value for the turbulent curves in Fig. 10) in Eq. 97 gives

$$G = \frac{V}{I_d q} = 2.67 \times 10^{-7} \frac{N_d N_g N_{dg}}{D_c^2}. \quad (102)$$

This value may be used for preliminary design calculations, even if the coil proportions deviate appreciably from those of Fig. 3. The 2.67 factor will be increased for more than four d-g coil sections and decreased for two d-g sections, the minimum possible for a balanced system. Even in these cases, the 2.67 factor is probably close enough for preliminary estimates.

The high signal levels provided by the eddy-current flowmeter are demonstrated by estimates on a model for a 15/16-in. (0.0238-m) channel diameter. This model has  $N_d = N_g = 550$ , and  $N_{dg} = 4$ . Substituting these values in Eq. 102 gives

$$\frac{V}{I_d q} = 570 \text{ V/A}/(\text{m}^3/\text{sec}) \quad (103)$$

or

$$\frac{V}{I_d q} = 36 \text{ mV/A/gpm.} \quad (104)$$

A current of 0.35 A causes only a small coil temperature rise and is supplied at a convenient power level. Thus, the sensitivity of the flowmeter at this nominal current is

$$\frac{V}{q} = 12.6 \text{ mV/gpm.} \quad (105)$$

## CONCLUSIONS

1. The eddy-current flowmeter appears well suited to measurements in high-temperature liquid-metal systems since it is not subject to curie-point limitations and temperature and radiation effects on permanent magnets and other ferromagnetic materials.
2. The configuration in which a system of balanced solenoids is wound around the outside of the flow channel is a high-signal-level device.
3. This configuration has an optimum operating frequency (depending on the channel diameter and fluid resistivity) which yields a maximum sensitivity and a signal voltage in phase with the primary current.
4. Temperature errors are minimized if an operating frequency is selected that is the optimum value for the fluid near the midpoint of the expected temperature range. Alternatively, a simple temperature compensation system is to allow the frequency to follow the fluid resistivity changes by adjusting for zero phase angle in the signal.

5. Operating frequencies much in excess of the optimum value produce an extreme sensitivity to fluid layers near the boundary surface and therefore create sensitivity uncertainties due to velocity-profile changes.

6. The optimum operating frequency for large-diameter channels becomes very low. If high speed of response is required in the flowmeter, the requirement for low frequency may limit the range of application of the external coil system to the smaller channel diameters.

## APPENDIX A

Program SIGPROF

```

* PROGRAM SIGPROF
C 6/18/68 D E WIEGAND 2919
C     SIGNAL PROFILE FUNCTION FOR EDDY-CURRENT FLOWMETER
C SURRTS BESEL, DESSEL, TOMECT, TOPOLR, BORDER, MOVEUP
C S DIVISOR NOT IN FUNCTION
1 FORMAT(1W1,/19X,1HS*18X,2HXS,16X,4HBESR,16X,4HBESI,16X,4HBESS,
116X,4HRESA/6E20.6//7X,13HRELATIVE AREA,19X,1HA,16X,4HSPFR,
114X,4HSPFI,16X,4HSPFS,16X,4HSPFA)
2 FORMAT(2F20.3,F20.6,F20.5)
3 FORMAT(5F20.6)
PI=3.1415927
NUMBS=8
NINCR=50
NINCRB=NINCR+1
SINIT=.25
SRATIO=2.
FINCR=NINCR
S=SINIT/SRATIO
REWIND 2
DO 99 N=1,NUMBS
S=S*SRATIO
XS=SQRTF(2.*S)
CALL BESEL(XS,BESR,BESI)
CALL TOPOLR(BESR,BESI,BESS,BESA)
PRINT 1,S,XS,BESR,BESI,BESS,BESA
DELTAA=S/FINCR
A=(-1.)*DELTAA
DO 90 M=1,NINCRB
A=A+DELTAA
X=SQRTF(2.*A)
CALL DESSEL(X,DBESR,DBESI)
CALL TOPOLR(DBESR,DBESI,DBESS,DBESA)
VPFS=DRESS**2/RESS**2
VPFA=DRESA**2.-RESA**2.*PI/2.
CALL TORECT(VPFR,VPFI,VPFS,VPFA)
CALL TOPOLR(VPFR,VPFI,VPFS,VPFA)
RAREA=(M-1)**.02
WRITE TAPE 2,RAREA,VPFR,VPFI,VPFS,VPFA
PA=A,.0009
RAREA=RAREA*.0009
90 PRINT 2,RAREA,PA,VPFR,VPFI,VPFS,VPFA
99 CONTINUE
REWIND 2
PAUSE 1
C READY PEN 2 INCHES IN FROM LEFT BORDER
PLOT SPFR
CALL BORDER(6.5,9.0,10,11)
P=PLOTF(.153846,-122222,1)
P=PLOTF(0.,-.10,2)
DO 299 N=1,NUMRS
DO 290 M=1,NINCRB
READ TAPE 2,X,Y,B,B,B
290 P=PLOTF(X,Y,4)
299 P=PLOTF(0.,-.10,3)
REWIND 2
CALL MOVEUP(15.)
PLOT SPF1
CALL BORDER(6.5,9.0,10,10)
P=PLOTF(.153846,,111111,1)
P=PLOTF(0.,-.4,2)
DO 399 N=1,NUMBS
DO 390 M=1,NINCRB

```

```
READ TAPE 2,X,B,Y,B,B
390 P=PLOTF( X,Y,4)
399 P=PLOTF(0.,-4,3)
REWIND 2
CALL MOVEUP(15.)
PLOT SPFS
CALL BORDER(6.5,9.0,18,10)
P=PLOTF(.153846,,111111,1)
P=PLOTF(0.,0.,2)
DO 499 N=1,NUMBS
DO 490 M=1,NINCRB
READ TAPE 2,X,R,B,Y,B
490 P=PLOTF(X,Y,4)
499 P=PLOTF(0.,0.,3)
REWIND 2
CALL MOVEUP(15.)
PLOT SPFA
CALL BORDER(6.5,9.0,18,14)
P=PLOTF(.153846,,777778,1)
P=PLOTF(0.,-3.5,2)
NINCRL=NINCR=1
DO 599 N=1,NUMBS
READ TAPE2,B,B,B,B,B
READ TAPE 2,X,B,B,B,B,Y
P=PLOTF(X,Y,3)
DO 590 N=1,NINCRL
READ TAPE 2,X,B,B,B,B,Y
590 P=PLOTF(X,Y,4)
599 P=PLOTF(0.,-3.5,3)
REWIND 2
CALL MOVEUP(15.)
REWIND 2
END
```

## APPENDIX B

Program FLOMGEN

```

PROGRAM FLOMGEN
C   VELOCITY PROFILE PER PROGRAM PROVEL 7/15/68
C   D E WIEGAND 7/16/68
C   VARYING S AND RE
DIMENSION TEXP(14),RE(14), UAOUM(14)
DIMENSION VTR(14),VTI(14),VTS(14),VTA(14)
1  FORMAT(1H1)
2  FORMAT(//)
4  FORMAT(/)
5  FORMAT(2X)
29 FORMAT(16E5.0)
41 FORMAT(7F11.7)
42 FORMAT(43X,2HS=,F9.5)
43 FORMAT(8X,2HRE,7X,2HVR,7X,2HVI,7X,2HVS,7X,2HVA,12X,2HRE,7X,
12HVR,7X,2HVI,7X,2HVS,7X,2HVA)
44 FORMAT(5X,7HLAMINAR,3F9.6,F9.5,7X,7HUNIFORM,3F9.6,F9.5)
45 FORMAT(2X,F10.4,3F9.6,F9.5,4X,E10.4,3F9.6,F9.5)
46 FORMAT(2X,2HRE/2X,7E16.8/2X,7E16.8)
47 FORMAT(2X,3HEXP/2X,7E16.8/2X,7E16.8)
48 FORMAT(2X,6H UAOUM/2X,7E16.8/2X,7E16.8)
31 FORMAT(5F16.8)
51 FORMAT(21I0,2E16.8)
NRE=14
NUMBS=7
NUMBS=43
NINCR=25
NINCR=250
SINIT=.2,
SINIT=.25
SRATIO=2.*{(1./6.)}
PUNCH 51,NRE,NUMBS,SINIT,SRATIO
FINCR=NINCR
READ 29,(RE(I),I=1,NRE)
PUNCH 31,(RE(I),I=1,NRE)
DO 62 I=1,NRE
UAOUM(I)=.655+.035*.43429448*LOGF(RE(I))
62 TEXP(I)=UAOUM(I)/(1.-UAOUM(I))
PRINT 1
PRINT 46,(RE(I),I=1,NRE)
PRINT 4
PRINT 47,(TEXP(I),I=1,NRE)
PRINT 4
PRINT 48,( UAOUM(I),I=1,NRE)
PRINT 1
S=SINIT/SRATIO
NP=0
DO 99 N=1,NUMBS
S=S*SRATIO
XS=SQRTF(2.*S)
CALL BESEL(XS,BESR,BESI)
CALL TOPOLR(BESR,BESI,BESS,BESA)
VUR#0.
VUI#0.
VLR#0.
VLI#0.
DO 60 I=1,NRE
VTR(I)=0.
60 VTI(I)=0.
DELTAA=S/FINCR
A=(-.5)*DELTAA
DO 90 M=1,NINCR
A=A+DELTAA

```

```

X=SORTF(2.*A)
CALL DFSEL(X,PDBESR,PDBESI)
FUNCR=2.*PDBESR*PDBESI
FUNC1=PDBESI**2-PDBESR**2
VUR=VUR+FUNC1
VUI=VUI+FUNC1
AOS=X**2/2./S
ULNUA=2.*1.-AOS
VLR=VLR+FUNC1*ULNUA
VLI=VLI+FUNC1*ULNUA
DO 63 I=1,NRE
UTOUA=(1.-AOS**TEXP(I))/UADUM(I)
VTR(I)=VTR(I)+FUNC1*UTOUA
VT(I)=VT(I)+FUNC1*UTOUA
63 CONTINUE
90 CONTINUE
CALL TOPOLR(VUR,VUI,VUS,VUA)
CALL TOPOLR(VLR,VLI,VLS,VLA)
DO 65 I=1,NRE
PVTR=VTR(I)
PVTI=VT(I)
CALL TOPOLR(PVTR,PVTI,PVTS,PVTA)
VTS(I)=PVTS
65 VTA(I)=PVTA
CORR=INCR*BESS**2
CORA=BESA*2,
VUS=VUS/CORR
VLS=VLS/CORR
VUA=VUA-CORA
VLA=VLA-CORA
DO 67 I=1,NRE
VTS(I)=VTS(I)/CORR
67 VTA(I)=VTA(I)-CORA
CALL TORECT(VUR,VUI,VUS,VUA)
CALL TORECT(VLR,VLI,VLS,VLA)
CALL TOPOLR(VUR,VUI,VUS,VUA)
CALL TOPOLR(VLR,VLI,VLS,VLA)
DO 69 I=1,NRE
CALL TORECT(VTR(I),VT(I),VTS(I),VTA(I))
69 CALL TOPOLR(VTR(I),VT(I),VTS(I),VTA(I))
IF(NP-3) 71,71,70
70 PRINT 1
NP=0
71 CONTINUE
PRINT 2
S=S*1.00000002
PRINT 42,S
PRINT 43
PRINT 5
PRINT 44,VLR,VLI,VLS,VLA,VUR,VUI,VUS,VUA
PUNCH 31,VLR,VLI,VUR,VUI
PRINT 45,(RE(I),VTR(I),VT(I),VTS(I),VTA(I),I=1,NRE)
PUNCH 31,(VTR(I),VT(I),I=1,NRE)
NP=NP+1
99 CONTINUE
END
.4E4 .1E5 .2E5 .4E5 +1E6 .2E6 ,4E6 .1E7 .2E7 .4E7 .1E8 .2E8 .4E8 ,1E9

```

S= .25000

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.007688	.040622	,041343	1.38373	UNIFORM	.010263	.061173	.062028	1.40456
,4000E 04	.009055	.050070	,050882	1.39187	,1000E 05	.009131	.050654	.051471	1.39244
,2000E 05	.009188	.051105	,051925	1.39239	,4000E 05	.009246	.051565	.052357	1.39335
,1000E 06	.009323	.052185	,053011	1.39399	,2000E 06	.009382	.052664	.053493	1.39448
,4000E 06	.009441	.053152	,053984	1.39500	,1000E 07	.009519	.053812	.054647	1.39570
,2000E 07	.009578	.054321	,055160	1.39625	,4000E 07	.009637	.054841	.055682	1.39683
,1000E 08	.009716	.055544	,056387	1.39752	,2000E 08	.009775	.056087	.056933	1.39824
,4000E 08	.009834	.056642	,057489	1.39888	,1000E 09	.009912	.057392	.058242	1.39977

S= .28061

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.009647	.045297	,046313	1.36094	UNIFORM	.012882	.068284	.069488	1.38433
,4000E 04	.011363	.055856	,057000	1.37009	,1000E 05	.011458	.056509	.057059	1.37073
,2000E 05	.011531	.057014	,058168	1.37123	,4000E 05	.011604	.057528	.058686	1.37175
,1000E 06	.011700	.058221	,059385	1.37246	,2000E 06	.011774	.058757	.059925	1.37302
,4000E 06	.011848	.059303	,060475	1.37359	,1000E 07	.011946	.060041	.061218	1.37439
,2000E 07	.012020	.060612	,061792	1.37501	,4000E 07	.012097	.061193	.062377	1.37665
,1000E 08	.012193	.061980	,063168	1.37653	,2000E 08	.012268	.062588	.063779	1.37723
,4000E 08	.012342	.063209	,064403	1.37795	,1000E 09	.012440	.064049	.065246	1.37895

S= .31498

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.012092	.050424	,051853	1.33542	UNIFORM	.016152	.076111	.077806	1.36167
,4000E 04	.014245	.062210	,063820	1.34559	,1000E 05	.014364	.062940	.064559	1.34641
,2000E 05	.014455	.065504	,065129	1.34697	,4000E 05	.014547	.064078	.065709	1.34755
,1000E 06	.014668	.064854	,066492	1.34835	,2000E 06	.014761	.065453	.067097	1.34898
,4000E 06	.014854	.066063	,067713	1.34952	,1000E 07	.014977	.066888	.068545	1.35051
,2000E 07	.015070	.067526	,069188	1.35121	,4000E 07	.015164	.068177	.069843	1.35193
,1000E 08	.015287	.069056	,070728	1.35292	,2000E 08	.015381	.069737	.071413	1.35370
,4000E 08	.015474	.070431	,072111	1.35451	,1000E 09	.015597	.071371	.073055	1.35563

S = .35355

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.015136	.056009	.058018	1.30685	UNIFORM	.020228	.084682	.087065	1.33631
,4000E 04	.017835	.069148	.071411	1.31837	,1000E 05	.017984	.069963	.072238	1.31916
,2000E 05	.016099	.070593	.072876	1.31981	,4000E 05	.018213	.071234	.073525	1.32047
,1000E 06	.018366	.072099	.074402	1.32136	,2000E 06	.018482	.072768	.075078	1.32206
,4000E 06	.018598	.073450	.075768	1.32279	,1000E 07	.018753	.074371	.076699	1.32379
,2000E 07	.018870	.075084	.077419	1.32457	,4000E 07	.018987	.075810	.078152	1.32538
,1000E 08	.019142	.075793	.079143	1.32649	,2000E 08	.019260	.077553	.079909	1.32737
,4000E 08	.019377	.078329	.080691	1.32828	,1000E 09	.019531	.079380	.081747	1.32953

S = .39685

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.018915	.062045	.064864	1.27488	UNIFORM	.025293	.094005	.097348	1.30795
,4000E 04	.022293	.076666	.079841	1.28781	,1000E 05	.022480	.077574	.080765	1.28872
,2000E 05	.022623	.078275	.081479	1.29943	,4000E 05	.022767	.079899	.082205	1.29016
,1000E 06	.022958	.079954	.083185	1.29117	,2000E 06	.023104	.080700	.083942	1.29196
,4000E 06	.023249	.081460	.084713	1.29277	,1000E 07	.023443	.082488	.085775	1.29389
,2000E 07	.023590	.083283	.086560	1.29477	,4000E 07	.023737	.084094	.087380	1.29568
,1000E 08	.023932	.085190	.088488	1.29693	,2000E 08	.024079	.086039	.089345	1.29791
,4000E 08	.024225	.086906	.090219	1.29893	,1000E 09	.024419	.088079	.091401	1.30034

S = .44545

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.023587	.068495	.072443	1.23914	UNIFORM	.031566	.104056	.108739	1.27626
,4000E 04	.027809	.084729	.089176	1.25365	,1000E 05	.028044	.085739	.090209	1.25467
,2000E 05	.028223	.086519	.091006	1.25547	,4000E 05	.028402	.087314	.091818	1.25629
,1000E 06	.028642	.088388	.092913	1.25742	,2000E 06	.028823	.089218	.093759	1.25831
,4000E 06	.029006	.090065	.094620	1.25922	,1000E 07	.029248	.091209	.095784	1.26048
,2000E 07	.029432	.092095	.096683	1.26146	,4000E 07	.029616	.092998	.097600	1.26248
,1000E 08	.029860	.094220	.098838	1.26389	,2000E 08	.030044	.095166	.099796	1.26499
,4000E 08	.030228	.096132	.100773	1.26614	,1000E 09	.030470	.097441	.102094	1.26772

S= .50000

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.029338	.075291	.080805	1.19922	UNIFORM	.039299	.114770	.121312	1.24089
.4000E 04	.034603	.093266	.099478	1.21551	.1000E 05	.034896	.094386	.100631	1.21666
.2000E 05	.035119	.095252	.101921	1.21755	.4000E 05	.035344	.096135	.102426	1.21848
.1000E 06	.035642	.097327	.103649	1.21974	.2000E 06	.035870	.098250	.104593	1.22074
.4000E 06	.036098	.099190	.105554	1.22176	.1000E 07	.036400	.100462	.106853	1.22317
.2000E 07	.036630	.101446	.107857	1.22428	.4000E 07	.036860	.102450	.108880	1.22542
.1000E 08	.037165	.103810	.110262	1.22700	.2000E 08	.037395	.104863	.111331	1.22824
.4000E 08	.037625	.105938	.112421	1.22953	.1000E 09	.037928	.107395	.113895	1.23130

S= .56123

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.036372	.082312	.089990	1.15470	UNIFORM	.048780	.126020	.135132	1.20147
.4000E 04	.042922	.102145	.110797	1.17298	.1000E 05	.043287	.103385	.112082	1.17427
.2000E 05	.043565	.104344	.113073	1.17527	.4000E 05	.043845	.105321	.114083	1.17631
.1000E 06	.044217	.106641	.115445	1.17773	.2000E 06	.044500	.107663	.116497	1.17885
.4000E 06	.044784	.108705	.117969	1.18000	.1000E 07	.045162	.110115	.119016	1.18158
.2000E 07	.045448	.111207	.120135	1.18282	.4000E 07	.045735	.112321	.121275	1.18411
.1000E 08	.046115	.113829	.122216	1.18588	.2000E 08	.046402	.114999	.124007	1.18727
.4000E 08	.046689	.116193	.125223	1.18871	.1000E 09	.047067	.117812	.126866	1.19071

S= .62996

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.044912	.089372	.100022	1.10512	UNIFORM	.060325	.137596	.150239	1.15761
.4000E 04	.053034	.111162	.123165	1.12554	.1000E 05	.053487	.112529	.124594	1.12708
.2000E 05	.053833	.115586	.125697	1.12821	.4000E 05	.054180	.114664	.126820	1.12937
.1000E 06	.054642	.116121	.128336	1.13097	.2000E 06	.054994	.117250	.129506	1.13222
.4000E 06	.055348	.118401	.130699	1.13351	.1000E 07	.055817	.119959	.132309	1.13529
.2000E 07	.056173	.121166	.133554	1.13668	.4000E 07	.056530	.122399	.134822	1.13812
.1000E 08	.057002	.124068	.136537	1.14011	.2000E 08	.057360	.125363	.137862	1.14167
.4000E 08	.057717	.126686	.139214	1.14329	.1000E 09	.058188	.128480	.141043	1.14553

S= .70710

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.055180	.096200	.110902	1.05001	UNIFORM	.074259	.149182	.166642	1.10893
,4000E 04	.065213	.120012	.196586	1.07304	,1000E 05	.065774	.121512	.1381/2	1.07466
,2000E 05	.066201	.122673	.139396	1.07592	,4000E 05	.066631	.123857	.140643	1.07723
,1000E 06	.067204	.125460	.142325	1.07902	,2000E 06	.067640	.126700	.143625	1.08043
,4000E 06	.068078	.127967	.144949	1.08138	,1000E 07	.068659	.129682	.146737	1.08387
,2000E 07	.069101	.131012	.148119	1.08543	,4000E 07	.069544	.132370	.149527	1.08705
,1000E 08	.070130	.134212	.151430	1.08928	,2000E 08	.070574	.135641	.152902	1.09104
,4000E 08	.071017	.137101	.154403	1.09236	,1000E 09	.071602	.139084	.156433	1.09537

S= .79370

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.067381	.102422	.142599	.98890	UNIFORM	.090896	.160327	.184301	1.05503
,4000E 04	.079714	.128271	.151022	1.01475	,1000E 05	.080405	.129908	.152778	1.01656
,2000E 05	.080932	.131176	.154134	1.01799	,4000E 05	.081462	.132471	.155514	1.01945
,1000E 06	.082169	.134223	.157377	1.02146	,2000E 06	.082706	.135581	.158816	1.02304
,4000E 06	.083247	.136968	.160282	1.02467	,1000E 07	.083965	.138847	.162261	1.02690
,2000E 07	.084510	.140305	.163791	1.02856	,4000E 07	.085057	.141796	.1653>0	1.03048
,1000E 08	.085782	.143818	.167458	1.03298	,2000E 08	.086330	.145388	.169088	1.03495
,4000E 08	.086879	.146995	.170750	1.03699	,1000E 09	.087602	.149177	.172997	1.03981

S= .89090

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.081661	.107542	.195032	.92134	UNIFORM	.110496	.170422	.203108	.99557
,4000E 04	.096734	.135372	.166382	.95035	,1000E 05	.097581	.137148	.1683>0	.99239
,2000E 05	.098227	.138524	.169816	.95398	,4000E 05	.098878	.139929	.171339	.99563
,1000E 06	.099744	.141833	.173395	.95789	,2000E 06	.100404	.143310	.174983	.99566
,4000E 06	.101068	.144820	.176600	.96148	,1000E 07	.101950	.146867	.178785	.98399
,2000E 07	.102621	.148457	.180473	.96596	,4000E 07	.103293	.150084	.182194	.96800
,1000E 08	.104185	.152293	.184520	.97081	,2000E 08	.104860	.154010	.186319	.97302
,4000E 08	.105535	.155769	.188153	.97532	,1000E 09	.106427	.158160	.190634	.97848

S= 1.00000

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.098062	.110937	.148065	.84692	UNIFORM	.133198	.178689	.222872	.93023
,4000E 04	.116354	.140600	.182501	.87947	,1000E 05	.117386	.142509	.184631	.88176
,2000E 05	.118173	.143990	.186275	.88355	,4000E 05	.118967	.145504	.187949	.88540
,1000E 06	.120024	.147558	.190208	.88793	,2000E 06	.120829	.149152	.191954	.88992
,4000E 06	.121639	.150784	.193732	.89197	,1000E 07	.122717	.152999	.196133	.89478
,2000E 07	.123536	.154721	.197989	.89700	,4000E 07	.124359	.156484	.199881	.89929
,1000E 08	.125450	.158882	.202438	.90244	,2000E 08	.126276	.160748	.204416	.90492
,4000E 08	.127104	.162662	.206432	.90749	,1000E 09	.128197	.165267	.209160	.91104

S= 1.12246

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.116458	.111866	.161482	.76529	UNIFORM	.158953	.184184	.243240	.85879
,4000E 04	.138469	.143095	.199123	.80182	,1000E 05	.139717	.145128	.201452	.80439
,2000E 05	.140669	.146707	.203250	.80640	,4000E 05	.141629	.148322	.205081	.80847
,1000E 06	.142909	.150516	.207553	.81131	,2000E 06	.143885	.152222	.209463	.81354
,4000E 06	.144867	.153970	.211408	.81584	,1000E 07	.146174	.156346	.214035	.81900
,2000E 07	.147169	.156196	.216066	.82149	,4000E 07	.148168	.160093	.218136	.82406
,1000E 08	.149494	.162676	.240734	.82759	,2000E 08	.150500	.164689	.223098	.83038
,4000E 08	.151507	.166756	.245305	.83327	,1000E 09	.152839	.169575	.228289	.83725

S= 1.25992

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.136479	.109513	.174985	.67621	UNIFORM	.187420	.185840	.263937	.78116
,4000E 04	.162702	.141900	.215888	.71721	,1000E 05	.164196	.144038	.218420	.72009
,2000E 05	.165338	.145702	.220376	.72235	,4000E 05	.166490	.147406	.222368	.72467
,1000E 06	.168026	.149725	.225056	.72786	,2000E 06	.169198	.151531	.227134	.73036
,4000E 06	.170379	.153383	.229250	.73295	,1000E 07	.171952	.155906	.232108	.73649
,2000E 07	.173149	.157874	.234318	.73928	,4000E 07	.174353	.159895	.236570	.74216
,1000E 08	.175953	.162652	.239615	.74613	,2000E 08	.177167	.164806	.241969	.74926
,4000E 08	.178385	.167020	.244371	.75250	,1000E 09	.179996	.170046	.247618	.75698

S= 1.41421

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.157444	.103065	.188178	.57961	UNIFORM	.217884	.182552	.284251	.69739
,4000E 04	.188320	.136045	.292321	.62561	,1000E 05	.190092	.138263	.235057	.62884
,2000E 05	.191447	.139991	.297170	.63137	.4000E 05	.192815	.141765	.239322	.63398
,1000E 06	.194641	.144183	.242227	.63756	.2000E 06	.196035	.146070	.244472	.64037
,4000E 06	.197441	.148010	.246759	.64327	.1000E 07	.199315	.150657	.249848	.64725
,2000E 07	.200743	.152727	.252237	.65038	.4000E 07	.202181	.154856	.254671	.65361
,1000E 08	.204092	.157768	.257962	.65807	.2000E 08	.205546	.160047	.260508	.66158
,4000E 08	.207004	.162396	.263104	.66522	.1000E 09	.208937	.165614	.266614	.67024

S= 1.58740

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.178318	.091836	.200577	.47558	UNIFORM	.249183	.173332	.303540	.60776
,4000E 04	.214187	.124689	.247838	.52719	,1000E 05	.216263	.126951	.250771	.53082
,2000E 05	.217852	.128717	.253037	.53366	.4000E 05	.219457	.130534	.255344	.53659
,1000E 06	.221602	.133017	.258459	.54050	.2000E 06	.223242	.134960	.260867	.54375
,4000E 06	.224897	.136962	.263320	.54700	.1000E 07	.227106	.139701	.266634	.55147
,2000E 07	.228792	.141848	.269196	.55498	.4000E 07	.230490	.144062	.271808	.55861
,1000E 08	.232751	.147099	.275339	.56351	.2000E 08	.234473	.149484	.278070	.56755
,4000E 08	.236203	.151947	.280855	.57154	.1000E 09	.238499	.155331	.284622	.57727

S= 1.78180

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.197723	.075435	.211624	.36447	UNIFORM	.279715	.157494	.321006	.51281
,4000E 04	.238764	.10305	.261768	.42237	,1000E 05	.241163	.109565	.264885	.42644
,2000E 05	.243001	.111336	.267293	.42952	.4000E 05	.244860	.113163	.269745	.43291
,1000E 06	.247348	.115668	.273057	.43741	.2000E 06	.249253	.117634	.275617	.44095
,4000E 06	.251177	.119664	.278226	.44450	.1000E 07	.253748	.122453	.281750	.44961
,2000E 07	.255714	.124647	.284476	.45355	.4000E 07	.257696	.126916	.287254	.45763
,1000E 08	.260340	.130038	.291010	.46324	.2000E 08	.262356	.132498	.293916	.46766
,4000E 08	.264385	.135047	.296879	.47224	.1000E 09	.267083	.138563	.300887	.47857

S= 2.00000

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.214030	.053944	.220724	.24689	UNIFORM	.307521	.134881	.335801	.41334
.4000E 04	.260208	.083877	.273593	.31133	.1000E 05	.262940	.086085	.276674	.31639
.2000E 05	.265037	.087822	.279208	.31997	.4000E 05	.267159	.089620	.281790	.32365
.1000E 06	.270004	.092094	.285278	.32871	.2000E 06	.272185	.094044	.287974	.33267
.4000E 06	.274391	.096066	.290722	.33677	.1000E 07	.277344	.098854	.294455	.34239
.2000E 07	.279605	.101055	.297307	.34681	.4000E 07	.281889	.103341	.30235	.35138
.1000E 08	.284941	.106500	.304193	.35758	.2000E 08	.287273	.108999	.307256	.36264
.4000E 08	.289622	.111599	.310380	.36779	.1000E 09	.292753	.115199	.314604	.37489

S= 2.24492

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.225558	.028052	.247296	.12373	UNIFORM	.330507	.106041	.347102	.31046
.4000E 04	.276587	.055072	.282017	.19654	.1000E 05	.279649	.057172	.285434	.20166
.2000E 05	.282003	.058832	.288074	.20557	.4000E 05	.284388	.060558	.290765	.20980
.1000E 06	.287592	.062946	.294400	.21547	.2000E 06	.290052	.064837	.297210	.21992
.4000E 06	.292544	.066806	.300075	.22451	.1000E 07	.295887	.069536	.303948	.23082
.2000E 07	.298451	.071702	.306943	.23577	.4000E 07	.301045	.073961	.309998	.24091
.1000E 08	.304519	.077100	.314128	.24797	.2000E 08	.307179	.079596	.317324	.25354
.4000E 08	.309865	.082205	.320584	.25931	.1000E 09	.313452	.085836	.324992	.26729

S= 2.51984

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.230843	-.000893	.230844	-.00386	UNIFORM	.346750	.072311	.354210	.20559
.4000E 04	.286175	.022295	.287042	.07775	.1000E 05	.289550	.024231	.290502	.08349
.2000E 05	.292149	.025771	.293284	.08798	.4000E 05	.294788	.027382	.296057	.09262
.1000E 06	.298338	.029624	.299805	.09897	.2000E 06	.301070	.031412	.302704	.10396
.4000E 06	.303842	.033284	.305659	.10910	.1000E 07	.307568	.035895	.309655	.11618
.2000E 07	.310433	.037981	.312748	.12174	.4000E 07	.313337	.040168	.315901	.12750
.1000E 08	.317234	.043226	.320166	.13542	.2000E 08	.320226	.045673	.323466	.14167
.4000E 08	.323253	.048244	.326833	.14815	.1000E 09	.327305	.051846	.331386	.15709

S= 2.82843

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.228932	-.030984	.291019	-.13452	UNIFURM	.354850	.035732	.356644	.10035
.4000E 04	.287786	-.012403	.288053	-.04307	.1000E 05	.291444	-.010683	.291640	-.03664
.2000E 05	.294267	-.009303	.294414	-.03150	.4000E 05	.297137	-.007849	.297241	-.02640
.1000E 06	.301008	-.005807	.301064	-.01929	.2000E 06	.303993	-.004165	.304022	-.01370
.4000E 06	.307029	-.002434	.307038	-.00792	.1000E 07	.311118	-.000000	.311118	-.00000
.2000E 07	.314270	-.001959	.314276	-.00623	.4000E 07	.317472	-.004028	.317498	.01268
.1000E 08	.321781	-.006944	.321856	-.02157	.2000E 08	.325096	-.009295	.325229	.02858
.4000E 08	.328459	-.011781	.328671	-.03585	.1000E 09	.332975	.015291	.333325	.04589

S= 3.17480

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.219625	-.060002	.227674	-.26659	UNIFURM	.354220	-.001210	.354223	-.00341
.4000E 04	.281042	-.046591	.284878	-.16428	.1000E 05	.284941	-.045131	.288493	-.15708
.2000E 05	.287956	-.043945	.291290	-.15144	.4000E 05	.291029	-.042683	.294142	-.14562
.1000E 06	.295182	-.040890	.298001	-.13765	.2000E 06	.298393	-.039433	.300988	-.13139
.4000E 06	.301665	-.037882	.304035	-.12492	.1000E 07	.306086	-.035678	.308158	-.11604
.2000E 07	.309501	-.033886	.313135	-.10905	.4000E 07	.312979	-.031977	.314609	-.11811
.1000E 08	.317673	-.029262	.319018	-.09135	.2000E 08	.321295	-.027051	.322432	-.08399
.4000E 08	.324978	-.024695	.325915	-.07584	.1000E 09	.329938	-.021338	.330628	-.06458

S= 3.56359

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.203558	-.085759	.220886	-.39873	UNIFURM	.345227	-.035971	.347096	-.10382
.4000E 04	.266486	-.077829	.27619	-.28415	.1000E 05	.270576	-.076662	.281226	-.27609
.2000E 05	.273746	-.075697	.284019	-.26978	.4000E 05	.276984	-.074655	.286869	-.26327
.1000E 06	.281372	-.073150	.290726	-.25434	.2000E 06	.284774	-.071908	.293713	-.24733
.4000E 06	.288249	-.070570	.296762	-.24010	.1000E 07	.292956	-.068641	.300890	-.23015
.2000E 07	.296604	-.067051	.304089	-.22232	.4000E 07	.300328	-.065340	.307354	-.21422
.1000E 08	.305369	-.062876	.311775	-.20306	.2000E 08	.309271	-.060846	.315199	-.19425
.4000E 08	.313249	-.058660	.318694	-.18512	.1000E 09	.318624	-.055513	.323424	-.17249

S= 4.00000

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.182123	-.106455	,210954	-.52896	UNIFORM	.329118	-.066350	,335740	-.19893
,4000E 04	.245503	-.104062	,266647	-.40091	,1000E 05	.249728	-.103210	,270215	-.39191
,2000E 05	.253012	-.102483	,272980	-.38485	,4000E 05	.256375	-.101680	,275802	-.37757
,1000E 06	.260944	-.100491	,279625	-.36750	,2000E 06	.264497	-.099487	,282559	-.35976
,4000E 06	.268135	-.098385	,285616	-.35157	,1000E 07	.273078	-.096766	,289716	-.34054
,2000E 07	.276921	-.095408	,292896	-.33179	,4000E 07	.280854	-.093923	,296143	-.32272
,1000E 08	.286195	-.091751	,300543	-.31023	,2000E 08	.290343	-.089934	,303953	-.30038
,4000E 08	.294585	-.087955	,307435	-.29014	,1000E 09	.300334	-.085064	,312149	-.27600

S= 4.48985

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.157224	-.120944	,198360	-.65570	UNIFORM	.307777	-.090815	,320896	-.28692
,4000E 04	.220059	-.124920	,292552	-.51286	,1000E 05	.224364	-.123392	,256056	-.50281
,2000E 05	.227720	-.122912	,298774	-.49494	,4000E 05	.231165	-.122359	,261551	-.48682
,1000E 06	.235859	-.121501	,265316	-.47558	,2000E 06	.239521	-.120749	,268237	-.46694
,4000E 06	.243281	-.119900	,271222	-.45790	,1000E 07	.248405	-.118614	,275271	-.44548
,2000E 07	.252401	-.117506	,278414	-.43571	,4000E 07	.256504	-.116270	,281625	-.42558
,1000E 08	.262093	-.114420	,285980	-.41152	,2000E 08	.266449	-.112842	,289359	-.40060
,4000E 08	.270916	-.111095	,292810	-.38916	,1000E 09	.276993	-.108500	,297485	-.37333

S= 5.03969

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.130950	-.128849	,183712	-.77730	UNIFORM	.283372	-.108650	,303488	-.36613
,4000E 04	.192360	-.136849	,236073	-.61835	,1000E 05	.196690	-.136644	,239497	-.60717
,2000E 05	.200077	-.136413	,242156	-.59841	,4000E 05	.203562	-.136111	,244875	-.58937
,1000E 06	.208327	-.135591	,248566	-.57697	,2000E 06	.212059	-.135097	,251433	-.56724
,4000E 06	.215894	-.134508	,254367	-.55718	,1000E 07	.221144	-.133568	,258350	-.54334
,2000E 07	.225252	-.132721	,261445	-.53245	,4000E 07	.229482	-.131747	,264611	-.52116
,1000E 08	.235265	-.130241	,268909	-.50593	,2000E 08	.239788	-.128919	,272247	-.49330
,4000E 08	.244441	-.127424	,275660	-.48053	,1000E 09	.250794	-.125151	,280287	-.46286

S= 5.65686

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.105266	-.130521	.167681	-.89209	UNIFORM	.258007	-.119933	.284520	-.43513
,4000E 04	.164506	-.143078	.218022	-.71584	,1000E 05	.168811	-.143186	.221359	-.70344
,2000E 05	.172189	-.143199	.223953	-.69373	,4000E 05	.175676	-.143144	.226610	-.68371
,1000E 06	.180458	-.142958	.230222	-.66996	,2000E 06	.184212	-.142722	.233032	-.65916
,4000E 06	.188090	-.142393	.235910	-.64799	,1000E 07	.193410	-.141802	.239824	-.63263
,2000E 07	.197589	-.141222	.242869	-.62054	,4000E 07	.201905	-.140515	.245988	-.60800
,1000E 08	.207827	-.139363	.250228	-.59070	,2000E 08	.212476	-.138308	.253526	-.57703
,4000E 08	.217275	-.137075	.256901	-.56282	,1000E 09	.223852	-.135143	.261483	-.54314

S= 6.34961

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.081770	-.126882	.150948	-.99832	UNIFORM	.233449	-.125380	.264988	-.49286
,4000E 04	.138235	-.143449	.199215	-.80390	,1000E 05	.142470	-.143856	.202465	-.79023
,2000E 05	.145804	-.144160	.204997	-.77952	,4000E 05	.149255	-.144284	.207593	-.76846
,1000E 06	.154005	-.144421	.211128	-.75329	,2000E 06	.157748	-.144435	.213883	-.74137
,4000E 06	.161625	-.144362	.216710	-.72904	,1000E 07	.166965	-.144115	.220560	-.71207
,2000E 07	.171175	-.143800	.223561	-.69870	,4000E 07	.175537	-.143361	.226640	-.68484
,1000E 08	.181545	-.142566	.230833	-.66570	,2000E 08	.186281	-.141780	.234099	-.65057
,4000E 08	.191185	-.140817	.237447	-.63483	,1000E 09	.197934	-.139234	.242000	-.61302

S= 7.12719

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.061559	-.119210	.194166	-1.09411	UNIFORM	.210978	-.126117	.245799	-.53877
,4000E 04	.114775	-.139197	.180414	-.88126	,1000E 05	.118899	-.139881	.183586	-.86629
,2000E 05	.122157	-.140342	.186060	-.85456	,4000E 05	.125540	-.140750	.188603	-.84245
,1000E 06	.130214	-.141194	.192072	-.82583	,2000E 06	.133910	-.141447	.194780	-.81276
,4000E 06	.137751	-.141620	.197564	-.79924	,1000E 07	.143062	-.141706	.201365	-.78063
,2000E 07	.147266	-.141650	.204333	-.76596	,4000E 07	.151636	-.141473	.207384	-.75073
,1000E 08	.157681	-.141029	.21548	-.72970	,2000E 08	.162465	-.140512	.214799	-.71306
,4000E 08	.167437	-.139817	.218138	-.69574	,1000E 09	.174309	-.138586	.222688	-.67172

S = 8.00000

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.045198	-.108919	,117925	-1.17744	UNIFORM	.191336	-.123449	.227704	-.5/298
,4000E 04	.094810	-.131715	,162289	-.94689	,1000E 05	.098788	-.132649	.165393	-.93067
,2000E 05	.101942	-.133310	,167820	-.91795	,4000E 05	.105227	-.133924	.170319	-.90482
,1000E 06	.109784	-.134653	,173735	-.88678	,2000E 06	.113402	-.135130	.176409	-.87260
,4000E 06	.117174	-.135535	,179163	-.85792	,1000E 07	.122413	-.135938	.182932	-.83770
,2000E 07	.126575	-.136128	,185882	-.82174	,4000E 07	.130919	-.136204	.188922	-.80518
,1000E 08	.136954	-.136102	,193081	-.78227	,2000E 08	.141751	-.135848	.196336	-.76413
,4000E 08	.146755	-.135416	,199687	-.74523	,1000E 09	.153704	-.134534	.204265	-.71898

S = 8.97970

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.032754	-.097375	,102737	-1.24630	UNIFORM	.174770	-.118662	.211247	-.59646
,4000E 04	.078521	-.122356	,145384	-1.00024	,1000E 05	.082322	-.123512	.148432	-.98290
,2000E 05	.085347	-.124351	,150822	-.96929	,4000E 05	.088509	-.125151	.153286	-.95523
,1000E 06	.092914	-.126138	,156665	-.93592	,2000E 06	.096425	-.126822	.159316	-.92072
,4000E 06	.100100	-.127440	,162053	-.90497	,1000E 07	.105226	-.128139	.165807	-.88327
,2000E 07	.109317	-.128562	,168755	-.86612	,4000E 07	.113602	-.128878	.171799	-.84831
,1000E 08	.119584	-.129102	,175976	-.82365	,2000E 08	.124361	-.129100	.179255	-.80409
,4000E 08	.129365	-.128924	,182639	-.78359	,1000E 09	.136347	-.128381	.187276	-.75531

S = 10.07937

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.023892	-.085744	,089011	-1.29904	UNIFORM	.161129	-.112873	.196727	-.61108
,4000E 04	.065681	-.112290	,190089	-1.04153	,1000E 05	.069280	-.113636	.133090	-1.02329
,2000E 05	.072157	-.114629	,135449	-1.00897	,4000E 05	.075175	-.115592	.137887	-.99417
,1000E 06	.079397	-.116809	,141238	-.97381	,2000E 06	.082778	-.117677	.143875	-.95776
,4000E 06	.086330	-.118489	,146603	-.94113	,1000E 07	.091308	-.119458	.150357	-.91817
,2000E 07	.095299	-.120096	,153313	-.90001	,4000E 07	.099498	-.120635	.156374	-.88112
,1000E 08	.105388	-.121166	,160586	-.85492	,2000E 08	.110115	-.121404	.163903	-.83411
,4000E 08	.115089	-.121474	,167336	-.81237	,1000E 09	.122069	-.121258	.172056	-.78208

S= 11.31371

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.017994	-.074898	.077030	-.1.33501	UNIFORM	.149968	-.106933	.184187	-.61942
.4000E 04	.055783	-.102407	.116614	-.1.07201	.1000E 05	.059163	-.103909	.119572	-.1.05320
.2000E 05	.061875	-.105030	.121902	-.1.03841	.4000E 05	.064733	-.106132	.124315	-.1.02310
.1000E 06	.068748	-.107546	.147642	-.1.00201	.2000E 06	.071979	-.108575	.130267	-.98537
.4000E 06	.075387	-.109558	.192990	-.96809	.1000E 07	.080187	-.110769	.136747	-.94420
.2000E 07	.084055	-.111602	.199715	-.92527	.4000E 07	.088142	-.112346	.142796	-.90553
.1000E 08	.093906	-.113162	.197051	-.87812	.2000E 08	.098557	-.113625	.150413	-.85629
.4000E 08	.103474	-.113926	.193902	-.83343	.1000E 09	.110408	-.114023	.15818	-.80150

S= 12.69921

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.014300	-.065368	.006914	-.1.35541	UNIFORM	.140719	-.101389	.173441	-.62435
.4000E 04	.048175	-.093276	.14982	-.1.09404	.1000E 05	.051323	-.094900	.107890	-.1.0703
.2000E 05	.053861	-.096124	.110186	-.1.06005	.4000E 05	.056546	-.097338	.112570	-.1.04451
.1000E 06	.060337	-.098916	.15866	-.1.02307	.2000E 06	.063402	-.100081	.118474	-.1.00610
.4000E 06	.066650	-.101212	.141186	-.98845	.1000E 07	.071247	-.102634	.124940	-.96398
.2000E 07	.074972	-.103640	.127915	-.94454	.4000E 07	.078927	-.104568	.131012	-.92423
.1000E 08	.084535	-.105644	.195303	-.89594	.2000E 08	.089086	-.106315	.138705	-.87334
.4000E 08	.093922	-.106833	.192249	-.84952	.1000E 09	.100782	-.107227	.147156	-.81637

S= 14.25438

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.012048	-.057352	.058603	-.1.36373	UNIFORM	.132784	-.096493	.164142	-.62841
.4000E 04	.042190	-.085158	.095036	-.1.11081	.1000E 05	.045100	-.086871	.097880	-.1.09193
.2000E 05	.047459	-.088171	.100133	-.1.07701	.4000E 05	.049963	-.089472	.102477	-.1.06150
.1000E 06	.053518	-.091180	.105727	-.1.04002	.2000E 06	.056407	-.092457	.108305	-.1.02298
.4000E 06	.059482	-.093710	.110994	-.1.00521	.1000E 07	.063859	-.095313	.114728	-.98049
.2000E 07	.067426	-.096469	.117697	-.96078	.4000E 07	.071231	-.097560	.120796	-.94013
.1000E 08	.076658	-.098869	.125107	-.91126	.2000E 08	.081089	-.099730	.128537	-.88812
.4000E 08	.085823	-.100449	.192120	-.86375	.1000E 09	.092581	-.101122	.137102	-.82945

S = 16.00000

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.010586	-.050775	.051866	-1.36525	UNIFORM	.125653	-.092259	.155886	-.633333
,4000E 04	.037251	-.078062	.086499	-1.12535	,1000E 05	.039925	-.079831	.089258	-1.10704
,2000E 05	.042103	-.081184	.091452	-1.09237	,4000E 05	.044425	-.082547	.093742	-1.07706
,1000E 06	.047738	-.084354	.096923	-1.05580	,2000E 06	.050445	-.085717	.099459	-1.03886
,4000E 06	.053340	-.087068	.102108	-1.02114	,1000E 07	.057484	-.088820	.105799	-.99639
,2000E 07	.060880	-.090104	.108743	-.97659	,4000E 07	.064521	-.091336	.111827	-.95577
,1000E 08	.069748	-.092852	.116131	-.92654	,2000E 08	.074043	-.093885	.119569	-.90301
,4000E 08	.078656	-.094788	.125173	-.87813	,1000E 09	.085287	-.095720	.128204	-.84297

S = 17.95940

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.009450	-.045387	.046361	-1.36521	UNIFORM	.118968	-.088548	.148304	-.63984
,4000E 04	.032943	-.071832	.079026	-1.14079	,1000E 05	.035387	-.073629	.081691	-1.12277
,2000E 05	.037386	-.075013	.083814	-1.10843	,4000E 05	.039528	-.076416	.086034	-1.09342
,1000E 06	.042599	-.078291	.089130	-1.07248	,2000E 06	.045121	-.079718	.091602	-1.05573
,4000E 06	.047833	-.081145	.094194	-1.03815	,1000E 07	.051735	-.083016	.097817	-1.01349
,2000E 07	.054952	-.084405	.100718	-.99368	,4000E 07	.058421	-.085757	.103766	-.97277
,1000E 08	.063431	-.087454	.108036	-.94328	,2000E 08	.067575	-.088639	.111460	-.91943
,4000E 08	.072052	-.089709	.115062	-.89412	,1000E 09	.078533	-.090882	.120113	-.89516

S = 20.15874

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.008383	-.040876	.0412/2	-1.36849	UNIFORM	.112533	-.085163	.141126	-.64782
,4000E 04	.029027	-.066249	.072329	-1.15784	,1000E 05	.031249	-.068050	.074882	-1.14031
,2000E 05	.033075	-.069446	.076920	-1.12630	,4000E 05	.035041	-.070869	.079058	-1.11159
,1000E 06	.037873	-.072784	.082049	-1.09099	,2000E 06	.040213	-.074255	.084444	-1.07445
,4000E 06	.042739	-.075736	.086965	-1.05702	,1000E 07	.046397	-.077699	.090497	-1.03248
,2000E 07	.049431	-.079173	.093337	-1.01257	,4000E 07	.052719	-.080623	.096330	-.99168
,1000E 08	.057501	-.082474	.100540	-.96195	,2000E 08	.061482	-.083795	.103931	-.93779
,4000E 08	.065810	-.085015	.107511	-.91204	,1000E 09	.072121	-.086409	.112552	-.87528

S= 22.62742

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.007301	-.036954	,097669	-1.37572	UNIFORM	.106292	-.081925	.134200	-.67665
,4000E 04	.025408	-.061111	,006183	-1.17676	,1000E 05	.027420	-.062895	.068613	-1.15967
,2000E 05	.029082	-.064287	,070559	-1.14596	,4000E 05	.030877	-.065713	.072606	-1.13153
,1000E 06	.033478	-.067646	,075477	-1.11123	,2000E 06	.035638	-.069141	.077785	-1.09487
,4000E 06	.037981	-.070658	,080219	-1.07757	,1000E 07	.041394	-.072686	.083646	-1.05311
,2000E 07	.044242	-.074225	,086410	-1.03328	,4000E 07	.047346	-.075755	.089334	-1.01220
,1000E 08	.051889	-.077736	,093463	-.98220	,2000E 08	.055699	-.079174	.096803	-.95772
,4000E 08	.059867	-.080528	,100543	-,93151	,1000E 09	.065989	-.082124	.105351	-.89390

S= 25.39842

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.006228	-.033422	,093998	-1.38654	UNIFORM	.100271	-.078721	.127480	-.66557
,4000E 04	.022087	-.056281	,004660	-1.19682	,1000E 05	.023901	-.058033	.062762	-1.18011
,2000E 05	.025406	-.059406	,064611	-1.16666	,4000E 05	.027039	-.060821	.066561	-1.15246
,1000E 06	.029417	-.062751	,069305	-1.13242	,2000E 06	.031402	-.064254	.071517	-1.11621
,4000E 06	.033566	-.065790	,073858	-1.09901	,1000E 07	.036736	-.067860	.077166	-1.07461
,2000E 07	.039398	-.069447	,079844	-1.05475	,4000E 07	.042315	-.071038	.082686	-1.03356
,1000E 08	.046615	-.073124	,086719	-1.00327	,2000E 08	.050245	-.074662	.089994	-.97844
,4000E 08	.054242	-.076133	,093480	-.95175	,1000E 09	.060161	-.077912	.098436	-.91325

S= 28.50876

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.005227	-.030173	,030622	-1.39924	UNIFORM	.094523	-.075508	.120980	-.67403
,4000E 04	.019094	-.051697	,055111	-1.21699	,1000E 05	.020724	-.053403	.057284	-1.20062
,2000E 05	.022082	-.054748	,059034	-1.18741	,4000E 05	.023561	-.056139	.060883	-1.17342
,1000E 06	.025726	-.058049	,063495	-1.15362	,2000E 06	.027542	-.059546	.065608	-1.13756
,4000E 06	.029532	-.061085	,067849	-1.12046	,1000E 07	.032464	-.063176	.071029	-1.09611
,2000E 07	.034941	-.064793	,073614	-1.07622	,4000E 07	.037671	-.066429	.076367	-1.05493
,1000E 08	.041722	-.068598	,080289	-1.02435	,2000E 08	.045167	-.070218	.083490	-.99918
,4000E 08	.048986	-.071791	,056911	-,97201	,1000E 09	.054687	-.073733	.091800	-.93263

S= 32.00000

RE	VR	VI	VS	VA	RE	VR	VI	VS	VA
LAMINAR	.004351	-.027171	,027517	-1.41201	UNIFORM	.089089	-.072297	.114733	-.68172
,4000E 04	.016452	-.047354	,050130	-1.23641	,1000E 05	.017910	-.049003	.052174	-1.24038
,2000E 05	.019131	-.050309	,053824	-1.20741	,4000E 05	.020467	-.051667	.055573	-1.19365
,1000E 06	.022428	-.053542	,058050	-1.17411	,2000E 06	.024083	-.055020	.060060	-1.15821
,4000E 06	.025904	-.056549	,062200	-1.14123	,1000E 07	.028604	-.058642	.065246	-1.11697
,2000E 07	.030899	-.060274	,067730	-1.09708	,4000E 07	.033442	-.061938	.070390	-1.07571
,1000E 08	.037243	-.064167	,074193	-1.04490	,2000E 08	.040500	-.065854	.077311	-1.01941
,4000E 08	.044135	-.067512	,080658	-.99180	,1000E 09	.049605	-.069598	.085467	-.95156

## APPENDIX C

Program FLOMTEMP

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PROGRAM FLOMTEMP
7/24/68 D E WIEGAND 2919
C LAMINAR, RE RANGE, UNIFORM FLOW
C VELOCITY PROFILE PER PROGRAM PROVEL
C SUBROUTINES BESEL, DESEL, TORECT, TOPOLR
C DIMENSION VTR( 7),VTI( 7),VTS( 7),VTA( 7)
C DIMENSION T(7),TS(7),RE400(11),TEXP(7),JAOU(7)
C DIMENSION DENSTY(7),VISCTY(7),RSTVTY(7),RE(7)
9 FORMAT(//)
10 FORMAT(1H1)
21 FORMAT(8E10.0)
22 FORMAT(1X,7E15.8,2X,6WTEMP,C/)
27 FORMAT(1X,7E15.8,2X,3WEXP/)
28 FORMAT(1X,7E15.8,2X,1WS/)
29 FORMAT(1A6.0)
31 FORMAT(5E16.8)
32 FORMAT(1X,11E10.2,2X,5WRE400//)
33 FORMAT(1X,5E11.5,5X,1HS,XS,BESS,BESA,EXP/
   1X,5E11.5,5X,22H JAOU(7),VTR,VTI,VTS,VTA/)
34 FORMAT(1X,6HRE400=,E14.8)
35 FORMAT(1X,3HT/C 3X,7E16.8)
36 FORMAT(1X,3HVR /3X,7E16.8)
37 FORMAT(1X,3HVI /3X,7E16.8)
38 FORMAT(1X,3HVS /3X,7E16.8)
39 FORMAT(1X,3HVA /3X,7E16.8)
40 FORMAT(1X,2HRE,4X,7E16.8)
41 FORMAT(1X,3HEXP,3X,7E16.8)
3 FORMAT(1X,7E15.8,2X,10HDENSTY,MKS/)
4 FORMAT(1X,7E15.8,2X,10HVISCTY,MKS/)
5 FORMAT(1X,7E15.8,2X,10HRSTVTY,MKS/)
8 FORMAT(1X,7E15.8,2X,1WS/)
50 FORMAT(21D,1E16.8)
60 FORMAT(1X,12H LAMINAR FLOW)
61 FORMAT(1X,16H UNIFORM VELOCITY)
      PRINT 10
      NINCR=25
      NINCR=250
      FINCR=NINCR
      NRE400=6
      NRE400=11
      NRPLUS=NRE400*2
      NT=7
      S400=2.*2.**(2./3.)
      GO TO 301
301 CONTINUE
      PUNCH 50,NT,NRE400,S400
      READ 21,(T(I),I=1,NT)
      PRINT 22,(T(I),I=1,NT)
      GO TO 302
302 CONTINUE
      PUNCH 31,(T(I),I=1,NT)
      DO 100 I=1,NT
C      EPSTEIN P 26 SODIUM NAK SUPPLEMENT 1 JULY 1955 MKS REVISION
C      DENSTY(I)=951.4+.2392*T(I)
C      ANDRADE P27 DITTO ABOVE
C      VISCTY(I)=1.142E-5*DENSTY(I)**.33333333*EXP(.7398*DENSTY(I) /
C      1(273.18*T(I)))
C      EPSTEIN P 29 DITTO C ABOVE
C      RSTVTY(I)=1.E=R*(10.892*.015272*T(I)+3.6746E-5*T(I)**2-379.26/T(I)
C      1)
100 CONTINUE
      PRINT 3,(DENSTY(I),I=1,NT)

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PRINT 4,(VISCTY(I),I=1,NT)
PRINT 5,(RSTVTY(I),I=1,NT)
DO 105 I=1,NT
105 TS(I)=S400*RSTVTY(4)/RSTVTY(I)
PRINT 8,(TS(I),I=1,NT)
READ 29,(RE400(J),J=1,NRE400)
PRINT 32,(RE400(J),J=1,NRE400)
GO TO 303
303 CONTINUE
PUNCH 31,(RE400(J),J=1,NRE400)
PRINT 10
NPAGE=0
DO 500 J=1,NRPLUS
DO 400 I=1,NT
IF(J=1) 250,252,250
250 IF(J=NRPLUS) 251,252,251
251 K=J-1
RE(I)=RE400(K)*DENSTY(I)/DENSTY(4)*VISCTY(4)/VISCTY(I)
UAOUM(I)=.655*.035*.43429448*LOGF(RE(I))
TEXP(I)=UAOUM(I)/(1.-UAOUM(I))
252 CONTINUE
VTR(I)=0.
VTI(I)=0.
S=TS(I)
XS=SQRTF(2.*S)
CALL BESEL(XS,BESR,BESI)
CALL TOPOLR(BESR,BESI,BESS,BESA)
DELTAA=S/FINCR
A=(-.5)*DELTAA
DO 90 M=1,NINCR
A=A+DELTAA
PX=SQRTF(2.*A)
CALL DESEL(PX,PDBESR,PDBESI)
FUNCR=2.*PDBESR*PDBESI
FUNC1=PDBESI**2-PDBESR**2
AOS=PX**2/2./S
IF(J=1) 253,254,253
253 IF(J=NRPLUS) 255,256,255
254 UTOUA=2.*(.1.*AOS)
GO TO 257
255 UTOUA=(1.-AOS**2*TEXP(I))/UAOUM(I)
GO TO 257
256 UTOUA=1.
257 CONTINUE
VTR(I)=VTR(I)+FUNCR*UTOURA
VTI(I)=VTI(I)+FUNC1*UTOURA
90 CONTINUE
CORR=FINCR*BESS**2
CORA=BESA**2.
CALL TOPOLR(VTR(I),VTI(I),VTS(I),VTA(I))
VTS(I)=VTS(I)/CORR
VTA(I)=VTA(I)-CORA
CALL TORECT(VTR(I),VTI(I),VTS(I),VTA(I))
CALL TOPOLR(VTR(I),VTI(I),VTS(I),VTA(I))
400 CONTINUE
IF(J=1) 258,259,258
258 IF(J=NRPLUS) 260,261,260
259 PRINT 60
PRINT 35,(T(I),I=1,NT)
GO TO 262
260 PRINT 34,RE400(K)
PRINT 35,(T(I),I=1,NT)

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```
PRINT 40,(RE(I),I=1,N#)
PRINT 41,(TEXP(I),I=1,NT)
GO TO 262
261 PRINT 61
PRINT 35,(T(I),I=1,NT)
262 CONTINUE
PRINT 36,(VTR(I),I=1,NT)
GO TO 306
306 CONTINUE
PUNCH31,(VTR(I),I=1,N#)
PRINT 37,(VTI(I),I=1,NT)
GO TO 307
307 CONTINUE
PUNCH31,(VTI(I),I=1,N#)
PRINT 38,(VTS(I),I=1,NT)
PRINT 39,(VTA(I),I=1,NT)
PRINT 9
NPAGE=NPAGE+1
IF(NPAGE=4) 236,236,237
237 PRINT 10
NPAGE=0
236 CONTINUE
500 CONTINUE
END
100.      200.      300.      400.      500.      600.      700.
.4E4 .1E5 .2E5 .4E5 .1E6 .2E6 .4E6 .1E7 .2E7 .4E7 .1E8
```

## APPENDIX D

Program PIPELOSS

```

* PROGRAM PIPELOSS
C FIELD ATTEN THRU THIN PIPE 4/12/68 D E WIEGAND RE 2919
1 FORMAT(1H1)
2 FORMAT(9X,1HS,8X,2HXS,6X,4HBESR,6X,4HBESI,6X,4HDESR,6X,4HDESI,
15X,5HFUNCR,5X,5HFUNC)
3 FORMAT(2F10.5,6F10.6)
4 FORMAT(/)
5 FORMAT(36H HC/HS=T*RHOF/RHOP/(RS*T/2.)*FUNC+1.)
6 FORMAT(41H HC,HS IS FIELD OUTSIDE, INSIDE PIPE WALL)
7 FORMAT(45H RHOF, RHOP IS RESISTIVITY OF FLUID, PIPE)
8 FORMAT(49H T IS WALL THICKNESS, RS IS INSIDE RADIUS OF PIPE)
NUMBS=43
SINIT=.25
SRATIO=2.**(1./6.)
PRINT 1
PRINT 2
DO 100 N=1,NUMBS
S=SINIT*SRATIO***(N-1)
XS=SQRRT(2.*S)
CALL BESEL(XS,BESR,BESI)
CALL DESSEL(XS,DESR,DESI)
CALL TOPOLR(BESR,BESI,BESS,BESA)
CALL TOPOLR(DESR,DESI,DESS,DESA)
FUNCS=XS*DESS/BESS
FUNCA=DESA-BESA
CALL TORECT(FUNCR,FUNCi,FUNCS,FUNCA)
SA=S*1.0000003
100 PRINT 3,SB,XS,BESR,BESI,DESR,DESI,FUNCR,FUNCi
PRINT4
PRINT 5
PRINT 6
PRINT 7
PRINT 8
END

```

S	XS	BESR	BESI	DESR	DESI	FUNC R	FUNC I
.25000	.70710	.996094	.124945	-.022092	.353093	.015513	.248707
.28061	.74915	.995079	.140231	-.025270	.373962	.019510	.278791
.31498	.79370	.993800	.157381	-.031239	.396030	.024524	.312406
.35355	.84089	.992189	.176623	-.037146	.419353	.030808	.349924
.39885	.89089	.990159	.198208	-.044169	.443988	.038675	.391737
.44544	.94387	.987602	.222417	-.052519	.469986	.048504	.438253
.50000	.99999	.984381	.249565	-.062445	.497396	.060761	.489883
.56123	1.05946	.980324	.280001	-.074244	.526256	.076006	.547030
.62996	1.12246	.975213	.314112	-.088266	.556592	.094905	.610064
.70710	1.18920	.968777	.352326	-.104929	.588412	.118241	.679294
.79370	1.25992	.960670	.395114	-.124726	.621698	.146917	.754932
.89089	1.33483	.950462	.442995	-.148241	.656392	.181941	.837045
.99999	1.41421	.937608	.496529	-.176163	.692390	.224410	.925505
1.12246	1.49830	.921427	.556324	-.209304	.729515	.275454	1.019934
1.25992	1.58740	.901040	.623022	-.248622	.767495	.336169	1.119659
1.41421	1.68579	.875433	.697297	-.295238	.805931	.407505	1.223687
1.58740	1.78179	.843198	.779832	-.350462	.844251	.490138	1.330717
1.78179	1.88774	.802667	.871295	-.415819	.881650	.584327	1.439216
1.99999	1.99999	.751734	.972290	-.493066	.917013	.689790	1.547553
2.24492	2.11892	.687772	1.083301	-.584219	.948809	.805627	1.654209
2.51984	2.24492	.607517	1.204584	-.691557	.974959	.930343	1.758027
2.82842	2.37841	.506929	1.336037	-.817614	.992654	1.061972	1.858459
3.17480	2.51984	.381033	1.476987	-.965147	.998126	1.198326	1.955751
3.56359	2.66967	.223738	1.625906	-.1137045	.986345	1.337306	2.051011
3.99999	2.82842	.027656	1.779993	-.1,336167	.950641	1.477232	2.146133
4.48984	2.99661	-.216070	1.934601	-.1,565059	.882219	1.617100	2.243598
5.03967	3.17480	-.517894	2.082426	-.1,825480	.769565	1.756742	2.346171
5.65684	3.36358	-.889869	2.212400	-.2,117654	.597736	1.896845	2.456590
6.34959	3.56359	-.1,345427	2.308183	-.2,439091	.347537	2.038839	2.577272
7.12718	3.777549	-.1,898747	2.346139	-.2,782808	-.005344	2.184704	2.710101
7.99999	3.99999	-.2,563408	2.292691	-.3,134649	-.491130	2.336714	2.856309
8.97968	4.23784	-.3,349863	2.100957	-.3,469360	-.1,146411	2.497163	3.016467
10.07935	4.48984	-.4,261012	1.706664	-.3,744925	-.2,013764	2.668121	3.190573
11.31369	4.75682	-.5,284775	1.023563	-.3,894616	-.3,139656	2.851241	3.378236
12.69919	5.03968	-.6,382109	-.060988	-.3,816174	-.4,569108	3.047671	3.578906
14.25436	5.33935	-.7,468244	-1.688718	-.3,357742	-.6,334570	3.258064	3.792131
15.99997	5.65684	-.8,384319	-4.028174	-.2,300904	-.8,434937	3.482706	4.017764
17.05936	5.99322	-.8,856162	-7.261407	-.3,42896e+10	.798431	3.721719	4.256075
20.15870	6.34959	-.8,437563	-11.548556	2,915164e+13	.220211	3.975266	4.507753
22.62738	6.72716	-.8,438510	-18.951472	7,967676e+15	.262534	4.243723	4.773811
25.39837	7.12718	-.1,847838	-23.285936	15.343221e+16	.103760	4.527746	5.055433
28.50871	7.55098	6.719197	-29.856890	25.465654e+14	.327036	4.828257	5.353833
31.99995	7.99999	20.973624	-35.016671	38.311096	-7.660485	5.146350	5.670180

HC/Hs=T\*Rhof/Rhop/(rs+t/2)\*func\*1.

HC/Hs IS FIELD OUTSIDE, INSIDE PIPE WALL  
 Rhof, Rhop IS RESISTIVITY OF FLUID, PIPE  
 T IS WALL THICKNESS, RS IS INSIDE RADIUS OF PIPE

## APPENDIX E

Subroutines

```

SUBROUTINE TO RECT(CR,CI,CS,CA)
C 8/16/67 D E WIEGAND RE 2919
C CONVERTS POLAR COORDINATES CS SIZE, CA ANGLE TO RECTANGULAR
C COORDINATES CR REAL, CI IMAGINARY
CR=CS*COSF(CA)
CI=CS*SINF(CA)
END

SUBROUTINE TO POLR(CR,CI,CS,CA)
C 8/16/67 D E WIEGAND RE 2919
C CONVERTS RECTANGULAR COORDINATES CR REAL, CI IMAGINARY
C TO POLAR COORDINATES CS SIZE, CA ANGLE
PI=3.1415927
CS=SQRTF(CR**2+CI**2)
IF(CR)3,5,4
3 CA=ATANF(CI/CR)+PI
GO TO 18
4 CA=ATANF(CI/CR)
GO TO 21
5 IF(CI)6,7,8
6 CA=(-1.0)*PI/2.0
GO TO 21
7 CA=0.0
GO TO 21
8 CA=PI/2.0
GO TO 21
18 IF(CA-PI)21,21,19
19 CA=CA-PI*2.0
21 END

SUBROUTINE BESSEL(X,BER,BEI)
C 2/29/68 WEE LIMITS D E WIEGAND 2919
C POLYNOMIAL APPROX P384 ABRAMOWITZ AND STEGUN
C DIMENSION C(7)
C(1)=-64.0
C(2)=113.77778
C(3)=-32.3635
C(4)=2.6419140
C(5)=-.08349609
C(6)=.00122552
C(7)=-.000000901
WEF=.1E-37
U=(X/8.0)**2
UDEXP=U**2
BER=1.0
TERM=1.0
UEXP=1.0
DO 9 I=1,7
UEXP=UEXP*UDEXP
IF(UEXP-WEE) 7,7,8
7 I=7
TERM=0.
GO TO 9
8 TERM=C(I)*UEXP
9 BER=BER+TERM
C(1)=16.0
C(2)=-113.77778
C(3)=72.817777
C(4)=-10.547658
C(5)=.52185615
C(6)=-.01103667
C(7)=.00011346

```

```

BEI=C(1)*U
UEXP=U
DO 19 I=2,7
UEXP=UEXP*UDEXP
IF(UEXP=WEE) 17,17,18
17 I=7
TERM=0.
GO TO 19
18 TERM=C(I)*UEXP
19 BEI=BEI+TERM
END

SUBROUTINE DESEL(X,DBEDXR,DBEDXI)
C 2/29/68 WEE LIMITS D E WIEGAND 2919
C DERIVATIVES RESPECT TO X
C POLYNOMIAL APPROX P384 ABRAMOWITZ AND STEGUN
C DIMENSION C(7)
C(1)=-4.0
C(2)=14.222222
C(3)=-6.0681481
C(4)=.66047849
C(5)=-.02609253
C(6)=.00045957
C(7)=-.00000394
WEE=.1E-37
U=(X/8.0)**2
UDEXP=U**2
SUM=C(1)*U
UEXP=U
DO 9 I=2,7
UEXP=UEXP*UDEXP
IF(UEXP=WEE) 7,7,8
7 I=7
TERM=0.
GO TO 9
8 TERM=C(I)*UEXP
9 SUM=SUM+TERM
DBEDXR=X*SUM
C(1)=-10.666667
C(2)=11.377778
C(3)=-2.3116751
C(4)=.14477204
C(5)=-.00379386
C(6)=.00004609
C(7)=0.0
SUM=.50
TERM=1.0
UEXP=1.0
DO 19 I=1,6
UEXP=UEXP*UDEXP
IF(UEXP=WEE) 17,17,18
17 I=6
TERM=0.
GO TO 19
18 TERM=C(I)*UEXP
19 SUM=SUM+TERM
DBEDXI=X*SUM
END

```

```

SUBROUTINE BORDER(PEN,DRUM,NPEN,NDRUM)
FNP=NPNP
FND=NDRUM
DA=PEN/FNP
DB=DRUM/FND
P=PLOT(F(1.0,1.0,1))
P=PLOT(F(0.0,0.0,2))
P=PLOT(F(0.0,0.0,3))
DO 512 I=1,NPEN
FI=I
A=FI*DA
P=PLOT(F(A,0.0,4))
P=PLOT(F(A,0,1,4)
512 P=PLOT(F(A,0,0,4)
DO 513 I=1,NDRUM
FI=I
B=FI*DB
P=PLOT(F(PEN,B,4)
PER=PEN-.1
P=PLOT(F(PER,B,4)
513 P=PLOT(F(PEN,B,4)
DO 514 I=1,NPEN
FI=I
A=PEN-FI*DA
P=PLOT(F(A,DRUM,4)
DRUR=DRUM-.1
P=PLOT(F(A,DRUR,4)
514 P=PLOT(F(A,DRUM,4)
DO 515 I=1,NDRUM
FI=I
B=DRUM-FI*DB
P=PLOT(F(0.0,B,4)
P=PLOT(F(0.1,B,4)
515 P=PLOT(F(0.0,B,4)
P=PLOT(F(0.0,0.0,3)
END

SURROUTINE MOVEUP(HOWFAIR)
P=PLOT(F( 1.,1.,1)
P=PLOT(F(0.,0.,2)
P=PLOT(F(0.,HOWFAIR,3)
END

SUBROUTINE BORLOG(PEN,DRUM,NPEN,NDRUM)
C E WIEGAND RE DIV 2919 9/14/67
FNP=NPNP
FND=NDRUM
DRUR=DRUM-.1
PENR=PEN-.1
P=PLOT(F( 1.,1.,1)
P=PLOT(F(0.,0,2)
P=PLOT(F(0.,0,3)
IF(NPEN) 40,40,5
5 DO 39 I=1,2
DO 29 N=1,NPEN
FN=N
DO 19 M=1,10
FM=M
X=.43429448*PEN/FNP*LOG(FM)+PEN*(FN-1.0)/FNP
P=PLOT(F(X,,0,3)
P=PLOT(F(X,,1,4)
19 P=PLOT(F(X,,0,4)

```

```
29 CONTINUE
  IF(I=1) 38,38,39
38 P=PLOTF(.0,DRUMR,3)
  P=PLOTF(.0,,0,2)
39 P=PLOTF(.0,,0,3)
  P=PLOTF(.0,DRUMR,2)
  P=PLOTF(.0,,0,3)
40 CONTINUE
  IF(NDRUM) 90,90,45
45 DO 79 I=1,2
  DO 69 N=1,NDRUM
  FN=N
  DO 59 M=1,10
  FM=M
  Y=.43429448*DRUM/FND*LOGF(FM)+DRUM*(FN-1,0)/FND
  P=PLOTF(.0,Y,3)
  P=PLOTF(.1,Y,4)
59 P=PLOTF(.0,Y,4)
69 CONTINUE
  IF(I=1) 78,78,79
78 P=PLOTF(PENR,.0,3)
  P=PLOTF(.0,,0,2)
79 P=PLOTF(.0,,0,3)
  P=PLOTF(PENR,.0,2)
  P=PLOTF(.0,,0,3)
90 CONTINUE
END
```

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